Estimating the Elasticity of Intertemporal Substitution
Using Mortgage Notches*

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Abstract

Using a novel source of quasi-experimental variation in interest rates, we develop a new approach to estimating the Elasticity of Intertemporal Substitution (EIS). In the UK, the mortgage interest rate features discrete jumps — notches — at thresholds for the loan-to-value (LTV) ratio. These notches generate large bunching below the critical LTV thresholds and missing mass above them. We develop a dynamic model that links these empirical moments to the underlying structural EIS. The average EIS is small, around 0.1, and quite homogeneous in the population. This finding is robust to structural assumptions and can allow for uncertainty, a wide range of risk preferences, portfolio reallocation, liquidity constraints, and optimization frictions. We also use our model to characterize the link between the EIS and the reduced-form borrowing elasticities often estimated in the literature. This analysis demonstrates that reduced-form elasticities are in general not informative of the underlying structural parameter as the translation between the two is mediated by endogenous variables that can vary widely across borrowers.

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1 Introduction

How responsive are households to changes in the intertemporal price of consumption? In standard economic models this response is governed by the Elasticity of Intertemporal Substitution (EIS). The EIS is arguably one of the most important parameters in economics as it plays a central role for a range of questions in macro, public finance, household finance, and asset pricing. Unfortunately, there exists no consensus on a reasonable value for this parameter due to limitations in data and research designs. The most cited estimates in the literature range between 0 and 2, which is an enormous range in terms of its implications for intertemporal behavior and policy.

A fundamental difficulty in addressing this question is how to find exogenous variation in interest rates. Most studies rely on time series movements in interest rates, which are gradual and almost certainly endogenous to unobserved factors that affect consumption. Our starting point is a novel source of quasi-experimental variation in interest rates arising from the fact that UK banks offer *notched* mortgage interest schedules. That is, the mortgage interest rate features discrete jumps at critical thresholds for the loan-to-value (LTV) ratio. For example, the interest rate increases by almost 0.5pp on the entire loan when crossing the 80% LTV threshold. This creates very strong incentives to reduce leverage to a level below the notch, thereby giving up consumption today in order to get a lower interest rate and more consumption in the future. We develop an approach that allows us to uncover the EIS using bunching at such interest notches.

Our study is based on administrative mortgage data from the Financial Conduct Authority. The data cover the universe of household mortgages in the UK between 2008-2014, including rich information on mortgage contracts and borrower characteristics. The majority of UK mortgage products carry a relatively low interest rate for a period of 2-5 years after which a much higher reset rate kicks in, creating strong incentives to refinance at the time the reset rate starts to apply. This makes refinancing a common occurrence in the UK. We focus on the population of refinancers, because they allow for a clean assessment of borrowing and intertemporal consumption choices. Specifically, because housing choices are pre-determined for refinancers, estimating LTV responses in this sample allows us to isolate borrowing choices from housing choices.

We document large bunching below notches and missing mass (“holes”) above notches in the LTV distribution. Such bunching represents direct evidence that borrowers respond to interest
rates. A recent literature in public economics has developed approaches to translate bunching moments into reduced-form price elasticities, mostly focusing on behavioral responses to taxes and transfers in static contexts (Saez 2010; Chetty et al. 2011; Kleven & Waseem 2013). It remains an open question whether these bunching-based elasticities have any structural or external validity, and whether their interpretation is robust to allowing for dynamics (Kleven 2016). In this paper we consider an inherently dynamic decision context and take the bunching literature in a more structural direction.

Translating bunching moments — or indeed any quasi-experimental moment — into structural parameters that can be used for out-of-sample prediction requires a structural model (see Kleven 2016; Einav et al. 2015, 2017). We develop two different approaches. The first approach is based on a two-period model with no uncertainty, no portfolio reallocation, no liquidity demand, and several other simplifying assumptions. The purpose of this model is to illustrate our conceptual approach and to provide the most transparent way of translating a bunching moment into the EIS. The second approach is based on a rich stochastic lifecycle model that relaxes many of the simplifying assumptions made in the baseline model. This model is much more realistic, but computationally more involved and thus more of a “black box”. We show that these two approaches give essentially the same answer: the observed bunching at interest notches is consistent with a small EIS, around 0.1. Furthermore, we present a battery of robustness checks and extensions of the stochastic lifecycle model, which confirm our finding that the EIS is small.

Why are our results robust to parametric assumptions? The amount of bunching at a notch is governed by an indifference condition for the marginal bunching individual. That is, individuals are willing to move down to the notch until the point where the lifetime utility of locating at the notch (and getting a lower interest rate) is equal to the lifetime utility of locating at the best unconstrained location above the notch (and paying a higher interest rate). This decision is primarily governed by the two features that affect relative utility in those two locations: the size of the interest jump at the notch and the curvature of utility around the notch as reflected by the EIS. Most other parameters of the model affect lifetime utility by roughly similar magnitudes on each side of the notch and therefore cancel out in the bunching decision.\footnote{While these other parameters are not separable in utility, the local nature of the bunching approach implies that they have almost the same effect above and below the notch.}

A qualification to this robustness argument is the possibility of optimization frictions that attenuate bunching. Some borrowers may be stuck at LTV ratios above the notch, not because of

\footnote{While these other parameters are not separable in utility, the local nature of the bunching approach implies that they have almost the same effect above and below the notch.}
true intertemporal preferences, but because they do not pay attention to or understand the incentives created by the notch. In this case, a naïve bunching approach to estimating the EIS would be downward biased. However, a key empirical advantage of notches is that the size of the hole above the notch is directly informative of optimization frictions. As shown by Kleven & Waseem (2013), it is therefore possible to correct for frictions using the size of the hole. We develop a structural extension of the Kleven-Waseem friction approach, which implies that our small EIS estimates are not obviously biased by optimization frictions.

Instead of estimating the structural EIS, a large literature focuses on the reduced-form elasticity of borrowing (or saving) with respect to the interest rate. We use our framework to characterize the link between such reduced-form elasticities and the structural EIS. Even when the EIS is zero, the reduced-form elasticity can be sizable due to the wealth effect of interest rate changes. We provide a simple formula showing that the relationship between the structural and reduced-form elasticities is mediated through a forward-looking loan-to-wealth (LTW) ratio, namely the current level of debt relative to the future level of gross wealth.\(^2\) Hence, the reduced-form elasticity and the LTW combine to provide a sufficient statistic for the EIS. We show that, without knowledge of the LTW, reduced-form elasticities are not informative of the structural EIS that could be used for out-of-sample prediction. A wide range of reduced-form elasticities can be consistent with a given EIS depending on the borrower population being analyzed.

Our paper contributes to three literatures. First, we contribute to a large structural literature studying intertemporal substitution in consumption, reviewed by Attanasio & Weber (2010). This literature estimates consumption Euler equations using either aggregate data (e.g., Hall 1988; Campbell & Mankiw 1989) or micro survey data (e.g., Zeldes 1989; Attanasio & Weber 1993, 1995; Vissing-Jørgensen 2002; Gruber 2013). Most of the literature has relied on time series movements in interest rates, producing a very wide range of estimates depending on the analysis sample and empirical specification.\(^3\) The main conceptual differences between our approach and this literature is that we use interest rate notches at a point in time as opposed to interest changes over time, and that our estimating equation is not a standard Euler equation due to the discontinuous nature of the notched incentive. Our EIS estimates are at the lower end of the spectrum provided by these non-experimental studies.

\(^2\)The relationship also depends on the levels of the interest rate and the discount factor, but the effects of these parameters tends to be second-order.

\(^3\)A methodological exception is Gruber (2013) who uses cross-sectional and time series variation in capital income tax rates to identify the EIS and obtains very large estimates of about 2.
Second, we contribute to a reduced-form literature studying borrowing responses to the after-tax interest rate. This literature includes a number of natural experiment studies using variation in the after-tax interest rate created by taxes, subsidies, and regulation (e.g. Follain & Dunsky 1997; Ling & McGill 1998; Dunsky & Follain 2000; Martins & Villanueva 2006; Jappelli & Pistaferri 2007; DeFusco & Paciorek 2017). The range of estimates is very wide, from a zero effect in Jappelli & Pistaferri (2007) to elasticities of about 1 in Dunsky & Follain (2000) and 1.5-3.5 in Follain & Dunsky (1997). Our main conceptual contribution to this literature is to clarify the relationship between reduced-form borrowing elasticities and the structural EIS, demonstrating that the former by itself is not very informative about the latter. The translation between the two parameters is mediated by additional (endogenous) variables that can vary widely across borrowers.

Third, we contribute to the recent bunching literature in public economics (as reviewed by Kleven 2016). Most of this literature has focused on static contexts and reduced-form estimation. By combining a bunching approach with dynamic structural estimation, our paper is related to recent work by Einav et al. (2015, 2017) who analyze bunching at a kink point in US Medicare. They argue that the choice of model is crucial when translating bunching into a parameter that can be used for out-of-sample prediction. In particular, they highlight the role played by frictions in the form of lumpiness and randomness in the choice variable used to bunch. This contrasts with our finding that the structural EIS (“out-of-sample prediction”) is robust to the modeling assumptions we make. This difference can be explained in part by a conceptual difference between kink-based and notch-based estimation. In the case of notches, the amount of friction is directly accounted using an observational moment — the amount of missing mass above the notch — as opposed to making parametric assumptions about such frictions.

The paper is organized as follows. Section 2 describes the institutional setting and data, Section 3 characterizes the link between bunching, reduced-form elasticities, and the EIS in our baseline two-period model, Section 4 presents empirical results using the baseline model, Section 5 develops and structurally estimates our full stochastic lifecycle model, and finally Section 6 concludes.

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4 Related to our empirical approach, DeFusco & Paciorek (2017) estimate leverage responses using an interest notch created by the conforming loan limit in the US, although their estimates do not separate mortgage demand from housing demand as we do here. Most importantly, they do not pursue the analysis of structural parameters, which is the main contribution of our paper.

5 This finding echoes insights from early calibration studies, which showed that a given value of the EIS can imply widely different, but typically much larger, savings elasticities depending on other calibrated parameters (Summers 1981; Evans 1983).

6 As discussed by Kleven (2016), this general insight echoes findings elsewhere in the bunching literature showing that the conversion of observed bunching (at kinks) into a structural elasticity is very sensitive to the assumed model of optimization frictions (e.g., Saez 1999; Chetty et al. 2010, 2011; Gelber et al. 2017).
2 Institutional Setting, Data and Descriptives

2.1 UK Mortgage Market

The UK mortgage market has several institutional features that facilitate our analysis. First, the interest rate on mortgage debt follows a step function with discrete jumps — notches — at certain LTV thresholds. There are interest rate notches at LTVs of 60%, 70%, 75%, 80%, and 85%.\(^7\) When a borrower crosses one of these thresholds, the interest rate increases on the entire loan. The thresholds apply to the LTV ratio at the time of loan origination; the interest rate does not change as amortization or house price growth gradually reduces the LTV. The size of the interest rate jump at a given threshold varies across product types and over time.\(^8\) The notches are very salient: daily newspapers display menus of interest rates by bank and LTV bracket, and the LTV thresholds feature very prominently when shopping for mortgages. For example, the mortgage websites of all the major banks show LTV brackets and interest rates for their different products up front.\(^9\)

Second, most UK mortgage products come with a relatively low interest rate for an initial period — typically 2, 3, or 5 years — after which a much larger (and variable) reset rate starts to apply. The notched interest rate schedule described above applies to the rate charged during the initial period of 2-5 years as opposed to the rate charged over the entire term of the mortgage (typically 25-35 years). The large and variable reset rate creates a very strong incentive to refinance at the end of the initial lower-rate period. This makes refinancing a frequent occurrence in the UK. In this paper we focus specifically on refinancers as this will allow us to isolate borrowing choices from housing choices, which is critical to assess intertemporal consumption substitution.

Third, while borrowers have a strong incentive to refinance no later than at the onset of the reset rate, the cost of early refinancing means that there is also a strong incentive to refinance no sooner

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\(^7\) There is in principle also an interest notch at 90%. However, very few banks offered mortgages above 90% after the financial crisis, implying that this threshold became a corner solution rather than a notch for most borrowers in our data. Our empirical analysis therefore focuses on the notches below 90%.

\(^8\) There is also some — but much less — variation in the size of notches across banks within product type and time. In particular, some banks do not feature certain notches at some points in time, but we show later that such no-notch observations represent a very small fraction of the data.

\(^9\) A broad question not addressed in this paper is why UK banks impose such notched interest rate schedules, a type of question that often arises in settings with notched incentive schemes (Kleven 2016). The traditional explanation for upward-sloping interest rate schedules is that the default risk is increasing in leverage, either due to increasing risk for each borrower or due to adverse changes in the mix of borrowers. However, under the reasonable assumption of smoothly increasing default rates, standard models predict smoothly increasing interest rates. While the UK practice of implementing the increasing interest rate schedule as a step function may not be second-best efficient in standard models, it may be explained — as with other types of notches — by the simplicity and salience of notches to banks and their customers. Our empirical analysis of these notches is implicitly based on the assumption that default rates (in the absence of notches) are smooth around the threshold.
than this time. Specifically, UK mortgage contracts feature large pre-payment charges, typically 5 or 10 percent of the outstanding loan, on borrowers who refinance before reset rate onset. The combination of penalizing reset rates and heavy pre-payment charges implies that households have strong incentives to refinance right around the end of the initial lower-interest period.

To confirm that households act on these refinancing incentives, Figure A.1 shows the distribution of time-to-refinance in our data. The distribution features large excess mass in refinancing activity around 2, 3, and 5 years, consistent with the fact that these are the most common timings of the penalizing reset rate. The lightly shaded bars indicate the fraction of households in each monthly bin who refinance “on time”, i.e. around the time their reset rate kicks in. These bars show that the majority of households refinance around reset rate onset and that this can explain the excess mass at 2, 3, and 5 years.\(^\text{10}\) Note that this graph represents clear evidence that borrowers respond to interest rate changes, but on a different margin (refinance timing) than what is our main focus (borrowing and consumption). What is more, the empirical patterns documented here imply that the time of refinancing is effectively locked in by the reset rate structure. This is helpful for ruling out selection issues from endogenous refinance timing in the analysis below.

### 2.2 Data

Our analysis uses a novel and comprehensive administrative dataset containing the universe of mortgage product sales in the UK.\(^\text{11}\) This Product Sales Database (PSD) is collected by the Financial Conduct Authority for regulatory purposes and has information on mortgage originations back to April 2005. This includes very detailed information on the mortgage contract such as the loan size, the date the mortgage became active, the valuation of the property, the initial interest rate charged, whether the interest rate is fixed or variable, the end date of the initial interest rate (the time at which the higher reset rate starts applying), whether the mortgage payments include amortization, and the mortgage term over which the full loan will be repaid. The data also include a number of borrower characteristics such as age, gross income, whether the income is solely or jointly earned, whether the borrower is a first-time buyer, mover or refinancer, and the reason for the refinance. There are also some characteristics of the property such as the type of dwelling and the number of rooms.\(^\text{12}\)

\(^{10}\)Indeed, we only observe 19% of households not refinancing on time.

\(^{11}\)The FCA Product Sales Data include regulated mortgage contracts, and therefore exclude other regulated home finance products such as home purchase plans and home reversions, and unregulated products such as second charge lending and buy-to-let mortgages.

\(^{12}\)Full details of the dataset can be found on the FCA’s PSD website.
While we observe the borrower’s LTV ratio, the PSD does not include information on product origination fees. These fees, while small relative to the loan size, can sometimes be rolled into the loan without affecting the LTV statistic used to determine the borrower’s interest rate. For example, it is possible to observe an actual LTV ratio of 75.01% where the borrower was still offered the product with a maximum LTV of 75%. In order to address this, we exploit information on all mortgage products (including origination fees) in the UK available from the organization MoneyFacts between 2008Q4 and 2014Q4. For a mortgage observation in the PSD we find the corresponding product in MoneyFacts based on the lender, the date of the loan, the interest rate offered, and the mortgage type. Where the interest rate paid accords with the lower LTV bracket on offer from the lender, but where their actual LTV is marginally above the threshold, we subtract the product fee from the loan value reported in the PSD. Inspecting these individuals, the additional loan amounts typically correspond precisely to the value of the fee, and so this adjustment places a large number of such individuals exactly at the notch. While this matching exercise reduces the sample, it is crucial for our methodology that the LTV ratio we use corresponds exactly to the one determining the actual interest rate.

Another useful feature of the PSD is that we are able to observe whether the household is refinancing. Using information about the characteristics of the property and the borrower, we can match-up refinancers over time to construct a panel. As described later, the panel structure allows us to implement a novel approach for estimating the counterfactual LTV distribution absent notches. The refinancer panel will therefore be the baseline data set for our analysis.

Table 1 shows a range of descriptive statistics in different samples. Column 1 includes the full sample of mortgages sold between 2008Q4 and 2014Q4 where we can exploit fee information from MoneyFacts. The full sample is large, with around 2.8 million observations. Column 2 shows how the properties of the sample change when we restrict attention to refinancers. The descriptive statistics are very similar, although the LTV and LTI ratios are slightly lower for refinancers as one would expect. Column 3 shows the descriptive statistics in the panel of refinancers that we use in the empirical analysis. In moving from column 2 to 3, we lose refinancers for whom we lack sufficient information on their previous loans as well as those we are not able to match-up over time. Our estimation sample still includes over 550,000 mortgages. Importantly, the descriptive statistics are very stable across the three columns, suggesting that our estimation sample (column

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13 See www.moneyfacts.co.uk.
14 For each homeowner we use the location of their house by 6-digit postcode (a code that covers a very small geographical area, around 15 homes on average) and the date of birth of the homeowner.
3) has similar average characteristics as the full population of borrowers.

2.3 Interest Rate Jumps at Notches

As described above, the UK mortgage market features discrete interest rate jumps at critical LTV thresholds, namely at 60%, 70%, 75%, 80%, and 85%. The first step of our analysis is to estimate the size of these interest rate notches. Unlike standard bunching approaches in which the discontinuity is the same across agents, in our setting the interest rate notch varies by bank, mortgage product, and the time of loan origination (all of which we observe). As we will show, notches do not depend on individual characteristics conditional on bank, product, and time, which is important for ruling out selection bias in the estimated interest notches. This is because the UK mortgage market works like a mortgage supermarket in which banks offer their interest rate schedule on a given product to all borrowers who meet their lending standards, as opposed to entering into individual negotiations that depend on idiosyncratic factors.

Our empirical analysis will be based on the average interest rate jump at each notch conditional on bank, product, and time. We estimate these interest rate jumps non-parametrically using the following regression:

\[ r_i = f(LTV_i) + \beta_1 \text{bank}_i + \beta_2 \text{variability}_i \otimes \text{duration}_i \otimes \text{month}_i + \beta_3 \text{repayment}_i + \beta_4 \text{term}_i + \nu_i \quad (1) \]

where \( r_i \) is the nominal mortgage interest rate for individual \( i \), \( f(\cdot) \) is a step function with steps at each 0.25pp of the LTV ratio, \( \text{bank}_i \) is a vector of bank dummies, \( \text{variability}_i \) is a vector of interest variability dummies (fixed interest rate, variable interest rate, capped interest rate, and “other”), \( \text{duration}_i \) is a vector of dummies for the duration of the initial low-interest period (the time until the reset rate kicks in), \( \text{month}_i \) is a vector of dummies for the month in which the mortgage was originated, \( \text{repayment}_i \) is a vector of dummies for the repayment type (interest only, capital and interest, and “other”), and \( \text{term}_i \) is a vector of dummies for the total term length. We denote by \( \otimes \) the outer product, so that the term variability \( \otimes \) duration \( \otimes \) month allows for each combination of interest rate variability and duration to have its own non-parametric time trend.

Figure 1 plots the conditional interest rate as a function of LTV based on specification (1). In each LTV bin we plot the coefficient on the LTV bin dummy plus a constant given by the predicted value \( E[\hat{r}_i] \) at the mean of all the other covariates (i.e., omitting the contribution of the LTV bin dummies). The figure shows that the mortgage interest rate evolves as a step function with
sharp jumps at LTV ratios of 60%, 70%, 75%, 80%, and 85%. These interest jumps are larger at LTV thresholds higher up in the distribution. At the two top thresholds, the annual interest rate increases by almost 0.5pp. Importantly, the interest rate schedule is flat between notches. This implies that, conditional on product and bank characteristics, the mortgage interest rate is almost fully determined by the LTV notches we exploit.

The flatness of the interest schedule between notches suggests that individual characteristics (that vary by LTV) have no effect on the mortgage interest rate. Figure A.2 in the appendix verifies this by controlling for the individual characteristics we observe (such as age, income, and family status) in the estimation of the interest schedule. The figure shows that the results are virtually unchanged. If observables such as age and income do not matter for the interest notches, it is difficult to imagine any unobservables that would matter. These results confirm the institutional context described earlier, namely that the UK mortgage market works as a mortgage supermarket in which a given type of product is offered at a given price, independently of who buys it.\textsuperscript{15}

When estimating the interest jumps from the coefficients on the LTV bin dummies in equation (1), we are holding all non-LTV mortgage characteristics constant on each side of the LTV threshold. For example, if a household is observed in a 5-year fixed rate mortgage (in a particular bank and month) just below the notch, we are asking how much higher the interest rate would have been for that same product just above the notch. In practice, if the household did move above the notch it might decide to re-optimize in some of the non-LTV dimensions — say move from a 5-year fixed to a 2-year fixed rate — and this would give a different interest rate change. However, not only are such interest rate changes endogenous, they are conceptually misleading due to the fact that the non-interest characteristics of the mortgage have value to the borrower and are priced into the offered interest rate. Our approach of conditioning on non-LTV characteristics when estimating the interest rate schedule is based on a no-arbitrage assumption: within a given LTV bin, if lower-interest rate products or banks are available, in equilibrium this must be offset by less favorable terms in other dimensions. In this case, the within-product interest rate jump around the threshold is the right measure of the true price incentive.

\textsuperscript{15}Moreover, the global interest estimations shown in Figures 1 and A.2 understate flatness between notches compared to the more precise local estimations used later. The locally estimated interest schedules are essentially completely flat. This implies that “donut hole” approaches in which we exclude observations in a range around the threshold when estimating the interest rate jump give virtually unchanged results.
2.4 Actual and Counterfactual LTV Distributions

Given the interest rate schedule in Figure 1, we expect to observe bunching just below notches and missing mass (holes) above notches in the LTV distribution. The idea of our approach is to use these empirical moments to identify the EIS. Figure 2 plots the observed LTV distribution for UK homeowners between 2008-14 around the different notches (depicted with vertical dashed lines). The figure distinguishes between the LTV distribution in the full sample (Panel A) and in the sample of refinancers (Panel B). It is clear that there is very large and sharp bunching below every notch along with missing mass above every notch, consistent with the strong incentives created by the interest rate jumps at those points. The refinancer distribution is naturally shifted to the left compared to the full distribution due to amortization. As described above, we will focus on the refinancer sample for which house values are pre-determined, because this allows us to isolate mortgage demand from housing demand.

To quantify the amount of bunching and missing mass in the observed LTV distribution, we need an estimate of the counterfactual LTV distribution — what the distribution would have looked like without interest rate notches — and the public finance literature has developed approaches to obtain such counterfactuals (see Kleven 2016). The standard approach is to fit a flexible polynomial to the observed distribution, excluding data around the notch, and then extrapolate the fitted distribution to the notch (Chetty et al. 2011; Kleven & Waseem 2013). However, this approach is not well-suited for our context: it is based on the assumption that notches affect the distribution only locally, which may be a reasonable assumption when there is only one notch or if the different notches are located very far apart. This is not satisfied in our setting in which we have many notches located relatively close to each other, and where Figure 2 suggests that most parts of the distribution are affected by notches. For example, it would be difficult to evaluate the counterfactual density at the 75% LTV notch using observations further down the distribution, say around 70%, because those observations are distorted by other notches.

To resolve this issue, we propose a new approach to assess the counterfactual distribution that exploits the panel structure of the refinancer data. Based on the LTV in the previous mortgage, the amortization schedule, and the house value at the time of refinance (which is assessed by the bank), we measure the new LTV \textit{before the refinancer has taken any action}. We label this the \textit{passive} LTV as it would be the LTV if the homeowner simply rolled over debt between the two mortgage contracts. We will base our estimate of the counterfactual LTV on the passive LTV with an adjustment that we describe below.
In Panel A of Figure 3 we compare the actual LTV distribution to the passive LTV distribution in the sample of refinancers. We see that the passive LTV distribution is smooth overall; unlike the actual LTV distribution it features no excess bunching or missing mass around notches. In general, the two distributions in Figure 3A may be different for two reasons: (i) behavioral responses to notches, and (ii) equity extraction or injection that would have happened even without notches. The second effect does not create bunching or missing mass, but it may smoothly shift the distribution. In this case, the passive LTV distribution would not exactly capture the counterfactual LTV distribution. To gauge the importance of such effects, we use information on equity extracted among households who do not bunch at notches. Figure A.3 shows that equity extracted among non-bunchers is positive through most of the passive LTV distribution (except at the very top) and has a smooth declining profile. We adjust the passive LTV distribution for non-bunching effects on LTV using the profile of equity extracted in Figure A.3. The assumption we are making is that the equity extraction profile among non-bunchers is a good proxy for the equity extraction profile in the full population of refinancers (including bunchers) in the counterfactual scenario without notches. We relax this assumption below.

Our estimate of the counterfactual LTV distribution is shown in Panel B of Figure 3. Comparing the actual and counterfactual LTV distributions provides clear visual evidence of bunching and missing mass around each notch. Notice that, except for the region below the bottom notch at 60%, the actual and counterfactual distributions never line up. This is because the actual distribution below each notch is affected by missing mass due to a notch further down. This implies that the standard approach to obtaining the counterfactual — fitting a polynomial to the observed distribution, excluding data right around the notch — would produce biased estimates in our context.

The assumption that the equity extraction profile among non-bunchers is a good proxy for the counterfactual equity extraction profile among bunchers raises potential concerns about selection. It is possible that bunchers are selected on variables that would impact their counterfactual equity extraction. A straightforward extension of our approach is to control for selection on observables: income, age, family status, and the number of past and future bunching episodes. The last of these covariates intends to capture the possibility that bunchers at time $t$ may be a selected sample of “optimizers” (thus bunching more at other times as well) while non-bunchers may be a selected sample of “passives”.\textsuperscript{16} We regress equity extracted among non-bunchers on these covariates and

\textsuperscript{16}However, it turns out that the number of previous/future bunching events is not excessively large for households
predict equity extraction for both bunchers and non-bunchers from this regression. This approach makes virtually no difference to any of our results. Hence, if selection were an issue for our equity extraction adjustment, it would have to come from unobservables that impact equity extraction and are uncorrelated with (and therefore not picked up by) the observables that we do control for.\footnote{A fundamental reason why the counterfactual distribution is very robust to the equity extraction adjustment is that we are adjusting a distribution — the passive LTV distribution — that is relatively flat. If the passive LTV distribution had been completely flat, any shift to the left or right would have precisely zero impact on the bunching estimation.}

Finally, the distributions shown above feature a small spike at an LTV of 65\%, although this threshold is not associated with an interest notch. This spike is most naturally explained by round-number bunching, a phenomenon observed across a wide range of settings (see Kleven 2016). If we do not adjust for round-number bunching, the amount of excess mass at interest notches (all of which are located at round numbers) would overstate the true response to interest rates. While we could adjust for round-number bunching using the observed spike at 65\%, a concern may be that round-number bunching is different in different parts of the LTV distribution. Instead, we deal with this issue by exploiting that some banks at some points in time do not feature a specific notch. This allows us to net out round-number bunching at a given notch using bunching at that same threshold in a no-notch subsample. As we show in Section 4, this adjustment has only a minor impact on our results.

3 A Simple Structural Model

Having shown that households respond to interest rate notches by bunching, we now develop a model to translate such bunching into an estimate of the EIS. We start by demonstrating our conceptual approach using a simple two-period model. This model illustrates in the simplest possible manner how the quasi-experimental bunching moments relate to the underlying structural EIS, and it clarifies how the structural EIS relates to the reduced-form borrowing (or saving) elasticities that have been studied elsewhere. In Section 5 we extend the analysis to a rich stochastic lifecycle model that relaxes many of the assumptions made here.
3.1 The Link Between Bunching and the EIS

We consider households who live for two periods (0 and 1) and have perfect foresight. They are homeowners and have chosen to remain in their current dwellings in both periods, but face a mortgage refinancing choice at time zero. As a baseline, assume that they can refinance at a constant gross borrowing rate equal to \( R \) (i.e., there is no notch).

The utility of consuming housing services \( H_t \) is separable from the utility of consuming non-durable goods \( c_t \), and households place no value on residual wealth (e.g., bequests) at the end of period 1. Households value non-housing consumption in any period \( t \) via a constant EIS function \( \sigma^{-1} c_t \sigma^{-1} \) and discount the future by a factor \( \delta \). Hence, the lifetime utility derived from non-housing consumption is given by \( \sigma^{-1} (c_0 \sigma^{-1} + \delta c_1 \sigma^{-1}) \).

The households receive an exogenous stream of income, \( y_t \) in period \( t \). They have initial net wealth \( W_0 \) equal to housing wealth net of any mortgage debt and net of any refinancing costs incurred in period zero. For simplicity, we assume that households hold no assets other than housing and have no liabilities other than the mortgage. The budget constraint in period 0 is therefore given by

\[
c_0 = y_0 + W_0 - (1 - \lambda) P_0 H,
\]

where \( \lambda \) is the LTV of the new mortgage and \( P_0 H \) is housing value (using that \( H_0 = H_1 = H \)). The period-1 budget constraint is given by

\[
c_1 = y_1 - R \lambda P_0 H + (1 - d) P_1 H,
\]

where \( d \) is the rate of house depreciation.

Households choose consumption according to the standard Euler equation

\[
c_1 = (\delta R)^\sigma c_0.
\]

Equations (2)-(4) determine the choice of \( c_0, c_1, \) and \( \lambda \) as functions the exogenous parameters of the model. We note that the LTV choice \( \lambda \) is monotonically decreasing in initial wealth \( W_0 \) and in the interest rate \( R \).

To begin with, we simplify by assuming that households are heterogeneous only in \( W_0 \). Our general argument goes through if households are heterogeneous in other dimensions such as income, housing quality, or preferences. Below we analyze the important case where the EIS pa-
rameter itself is heterogeneous. If $W_0$ is smoothly distributed in the population, equations (2)-(4) imply a smooth density distribution of LTV, which we denote by $f_0(\lambda)$. We will refer to this as the counterfactual LTV distribution under a constant interest rate $R$. Our estimate of the empirical counterpart to $f_0(\lambda)$ was shown in Figure 3.

Suppose now that an interest rate notch is introduced at $\lambda^*$, so that the borrowing rate increases from $R$ to $R + \Delta R$ for LTVs exceeding $\lambda^*$. Figure 4 illustrates the implications of this notch for borrowing and consumption. Panel A depicts the period-1 budget constraint before and after the introduction of the notch in $\{\lambda, c_1\}$ space. It also shows the indifference curves before and after the notch for the marginal bunching household, i.e. the highest-LTV (lowest-wealth) household who will choose to bunch at the notch. When faced with the constant interest rate $R$, this household chooses an LTV of $\lambda^* + \Delta \lambda$, where the indifference curve is tangent to the initial budget constraint. After the introduction of the notch, this household is indifferent between locating at the LTV threshold $\lambda^*$ and locating at the best interior LTV $\lambda^I$, where the indifference curve is tangent to the notched budget constraint. All households whose LTV fell in the segment $[\lambda^*, \lambda^* + \Delta \lambda]$ before the notch was introduced are strictly better off bunching at the notch than at an interior LTV.

Panel B shows the LTV distribution before and after the introduction of the notch. In the presence of the notch, there is sharp bunching at $\lambda^*$ along with a hole in the distribution between $(\lambda^*, \lambda^I)$. The amount of bunching is equal to $B = \int_{\lambda^*}^{\lambda^* + \Delta \lambda} f_0(\lambda) d\lambda \simeq f_0(\lambda^*) \Delta \lambda$. Hence, with estimates of excess bunching $B$ and the counterfactual density around the notch $f_0(\lambda^*)$, it is possible to estimate the LTV response $\Delta \lambda$. The fundamental idea of our approach — a dynamic extension of Kleven & Waseem (2013) — is that we can use the indifference condition between $\lambda^*$ and $\lambda^I$ for the marginal buncher to derive a condition that relates the EIS $\sigma$ to the LTV response $\Delta \lambda$.

To characterize the estimating indifference condition, we first use that the marginal bunching household chooses the LTV ratio $\lambda^* + \Delta \lambda$ in the counterfactual scenario with a constant interest rate $R$. From equations (2)-(4), this allows us to relate initial wealth $W_0$ for this household to the other parameters of the model as follows

$$W_0 = P_0H - y_0 + \frac{y_1}{1 - d} \frac{P_1H - ((\delta R)^\sigma + R)(\lambda^* + \Delta \lambda) P_0H}{(\delta R)^\sigma}. \quad (5)$$

This relationship allows us to eliminate $W_0$ from the problem. This is helpful because our data do not contain information on non-housing assets and liabilities, and therefore do not enable us to measure total initial wealth.
Using wealth defined in equation (5) and the optimality conditions (2)-(4) evaluated at the interest rate $R + \Delta R$, we can solve for the lifetime utility of the marginal buncher at the best interior choice $\lambda^I$ in the presence of the notch. This is given by

$$V^I(\sigma, \delta, \Delta \lambda, \Delta R, x) = \frac{\sigma}{\sigma - 1} (P_0 H)^{\frac{\sigma - 1}{\sigma}} \left( \frac{\delta^\sigma (R + \Delta R)^{\sigma - 1} + 1}{(\delta R)^{\sigma - 1}} \right)^{\frac{1}{\sigma}} \times \left( \frac{(\delta R)^{\sigma}}{R + \Delta R} + 1 \right) \left( \frac{y_1}{P_0 H} + \Pi_1 \right) - ((\delta R)^{\sigma} + R)(\lambda^* + \Delta \lambda)^{\frac{\sigma - 1}{\sigma}},$$

where $\Pi_1 \equiv (1 - d) \frac{P_1}{P_0}$ is gross house price growth net of depreciation. In the indirect utility function $V^I(\cdot, \cdot)$, the argument $x$ is a vector that includes the parameters $\{\lambda^*, R, \frac{y_1}{P_0 H} + \Pi_1\}$.

Similarly, setting $\lambda = \lambda^*$ and applying the interest rate $R$, the budget constraints (2)-(3) and the wealth condition (5) allow us to evaluate lifetime utility at the notch as

$$V^N(\sigma, \delta, \Delta \lambda, x) = \frac{\sigma}{\sigma - 1} (P_0 H)^{\frac{\sigma - 1}{\sigma}} \left( \frac{1}{(\delta R)^{\sigma}} \left( \frac{y_1}{P_0 H} + \Pi_1 - R\lambda^* - ((\delta R)^{\sigma} + R)\Delta \lambda \right)^{\frac{\sigma - 1}{\sigma}} \right) + \delta \left( \frac{y_1}{P_0 H} + \Pi_1 - R\lambda^* \right)^{\frac{\sigma - 1}{\sigma}}. \quad (7)$$

The marginal buncher is indifferent between bunching at the notch and locating at the best interior LTV, allowing us to state the following proposition:

**Proposition 1.** Given a bunching moment $\{\Delta \lambda, \Delta R\}$ and a discount factor $\delta$, the EIS $\sigma$ is the solution to the indifference equation

$$F(\sigma, \delta, \Delta \lambda, \Delta R, x) \equiv V^N(\sigma, \delta, \Delta \lambda, x) - V^I(\sigma, \delta, \Delta \lambda, \Delta R, x) = 0,$$

where $x = \{R, \lambda^*, \frac{y_1}{P_0 H} + \Pi_1\}$, and where $V^I(\cdot)$ and $V^N(\cdot)$ are given by (6) and (7), respectively.

**Proof.** The proof is in Appendix A.1. \qed

Three points should be noted. First, the indifference equation contains an additional structural parameter $\delta$. In our EIS estimates, we use values of the discount factor $\delta$ from the existing literature and show that our results are robust to a large range of discount factors. Estimates are robust to values of $\delta$ because this parameter primarily affects the level of borrowing, whereas the EIS governs the response of borrowing to interest changes. Bunching responses to interest notches are of the latter form, and are therefore governed by the EIS and insensitive to $\delta$. Second, the only other value that requires calibration is $\frac{y_1}{P_0 H} + \Pi_1 = \frac{y_1 + (1-d)P_1 H}{P_0 H}$. This is a measure of future
resources from human wealth ($y_1$) and housing wealth ($(1-d) P_1 H$), scaled by current housing wealth. We estimate $\sigma$ using reasonable values of this variable in the following section.\textsuperscript{18} Third, even in this simplistic dynamic model, the estimating indifference equation (8) is considerably more involved than the static bunching estimator developed by Kleven & Waseem (2013). The static bunching estimator does not require calibrating any variables: the bunching moment maps directly into a structural elasticity. The added complexity of the dynamic approach increases by an order of magnitude when we turn to the full stochastic lifecycle model in Section 5. However, as we will show, it is a general feature of our methodology that many of the calibrated variables have a very small impact on the estimating indifference equation, making our results robust despite the analytical complexity of the expressions. The robustness of the approach results from the fact that the calibrated variables (such as future income and house prices) are invariant to the bunching decision and therefore affect $V^I$ and $V^N$ by roughly similar magnitudes.

The exposition above assumes that there is only one value of the structural EIS $\sigma$, while in practice there is likely to be heterogeneity in this parameter. In fact, the empirical LTV distributions shown in Section 2.4 suggest that this has to be the case: without heterogeneity, there would be a sharp hole in the LTV distribution between $\lambda^*$ and $\lambda^I$ as illustrated in Figure 4B, whereas the empirical LTV distribution in Figure 2 features a gradual hole and has some refinancers located just above the notch.\textsuperscript{19} This provides prima facie evidence that some households have very small $\sigma$s while others have larger $\sigma$s. As Kleven & Waseem (2013) and Kleven (2016) show, in the presence of heterogeneity in $\sigma$, our bunching approach estimates the average $\sigma$.

To see this, consider a general joint distribution of initial wealth $W_0$ and the EIS $\sigma$. At each elasticity level $\sigma$, households optimize as characterized above. In the counterfactual scenario with a constant interest rate $R$, there is a joint distribution of LTV and EIS given by $g_0(\lambda, \sigma)$ and an unconditional distribution of LTV given by $g_0(\lambda) = \int_\sigma g_0(\lambda, \sigma) \, d\sigma$. In the observed scenario with a notched interest rate, the marginal buncher at elasticity level $\sigma$ reduces LTV by $\Delta\lambda_{\sigma}$. We can then link bunching $B$ to the average LTV response at the notch $E[\Delta\lambda_{\sigma}|\lambda^*]$ as follows

\[
B = \int_\sigma \int_{\lambda^*}^{\lambda^* + \Delta\lambda_{\sigma}} g_0(\lambda, \sigma) \, d\lambda d\sigma \approx g_0(\lambda^*) E[\Delta\lambda_{\sigma}|\lambda^*],
\]

where the approximation assumes that the counterfactual density $g_0(\lambda, \sigma)$ is roughly constant in

\textsuperscript{18}In the full dynamic estimation in Section 5, expectations are calibrated within the model and we conduct a variety of robustness checks.

\textsuperscript{19}Besides very small $\sigma$s among some households, the presence of density mass just above the notch may reflect various optimization frictions (including liquidity constraints), an issue that we will address in Section 3.3.
λ on the bunching segment \((\lambda^*, \lambda^* + \Delta \lambda)\). In other words, in the presence of heterogeneous treatment effects, bunching identifies the local average treatment effect. When applying a bunching moment like \(E[\Delta \lambda|\lambda^*]\) to the estimating indifference equation (8), we are estimating EIS at the average LTV response as opposed to the average EIS. These two will in general be different due to the nonlinearity of (8), creating a form of aggregation bias. As elaborated by Kleven (2016), such aggregation bias is likely to be very small in practice.

### 3.2 The Link Between the EIS and the Reduced-Form Elasticity

As reviewed in the introduction, a large literature estimates reduced-form elasticities of borrowing or saving with respect to the interest rate. In contrast, our methodology allows us to estimate the structural EIS directly, without resorting to reduced-form estimates. How does one compare the magnitude of reduced-form elasticities to the EIS? We can use our framework to characterize the relationship between the two elasticity concepts. Denoting the elasticity of borrowing with respect to the interest rate by \(\varepsilon\), comparative statics on (2) to (4) give the following result.

**Proposition 2.** Given the EIS \(\sigma\), the discount factor \(\delta\), the gross interest rate \(R\), and the ratio \(LTW \equiv \frac{P_0 H - W_0 - y_0}{y_1 + (1 - d) P_1 H}\), the elasticity of borrowing with respect to the interest rate is given by

\[
\varepsilon = -\frac{\partial \log \lambda}{\partial \log R} = \frac{\sigma (\delta R)^\sigma + R}{(\delta R)^\sigma + R} - \frac{\sigma (\delta R)^\sigma \times LTW}{1 + (\delta R)^\sigma \times LTW}. 
\]  
(10)

**Proof.** The proof is in Appendix A.1.

Besides the interest rate \(R\) and the structural parameters \(\delta\) and \(\sigma\), the reduced-form elasticity depends on a ratio we have defined as \(LTW\). This ratio is inversely proportional to the borrower’s repayment capacity and will in general differ substantially across households. To get an intuitive sense of this ratio, consider a household whose only initial wealth is (the net worth of) housing and who has no current income. In that case, \(W_0 = (1 - \lambda_0) P_0 H\) and the ratio \(LTW = \frac{\lambda_0 P_0 H}{y_1 + (1 - d) P_1 H}\) represents a loan to future wealth ratio, with future wealth incorporating both human and financial (housing) wealth. For brevity, and with slight abuse of terminology, we therefore refer to this ratio as the loan-to-wealth ratio.

Figure 5 illustrates the mapping from the EIS to the reduced-form elasticity for a range of \(LTW\) ratios \((LTW = 0, \frac{1}{2}, \text{and } 1)\). This spans the case where mortgage debt has been fully repaid to the case where the household needs to devote its entire future resources (from human and financial
wealth) to debt servicing. The figure is presented for the case where $\delta \simeq R \simeq 1$, so that (10) simplifies to
\begin{equation}
\varepsilon \simeq \frac{1}{2} + \sigma \left( \frac{1}{2} - \frac{LTW}{1 + LTW} \right) .
\end{equation}
(11)

For a given $LTW$, the reduced-form elasticity is monotonically increasing in $\sigma$ (as $LTW \leq 1$). But the reduced-form elasticity can vary greatly with indebtedness for a given $\sigma$. As a result, when comparing estimates from samples with different degrees of indebtedness, it is not necessarily true that a higher $\varepsilon$ translates into a higher $\sigma$. Note also that in equation (11) the reduced-form elasticity is bounded from below by one-half (when $\sigma = 0$), because this is the magnitude of the wealth effect. This particular result is driven by the 2-period assumption. In appendix A.2, we generalize our results to a multi-period version of the baseline model (while retaining the other simplifying assumptions of this model).

Given that most studies estimate only reduced-form elasticities, it is interesting to flip equation (11) in order to express the structural EIS as a function of the observables $\varepsilon$ and $LTW$. This gives the following sufficient statistic formula for the EIS:

**Proposition 3.** If interest rates and discount rates are sufficiently low ($R \simeq \delta \simeq 1$), then the reduced-form elasticity $\varepsilon$ and the ratio $LTW \equiv \frac{P_0H - W_0 - y_0}{y_1 + (1-d)P_1H}$ combine to provide a sufficient statistic for the EIS given by
\begin{equation}
\sigma \simeq \frac{1 + LTW}{1 - LTW} (2\varepsilon - 1) .
\end{equation}
(12)

**Proof.** Follows from (11) with $R = \delta = 1$.

This formula provides guidance on interpreting the magnitude of estimated elasticities to researchers using reduced-form methods. It should be noted, however, that this “sufficient statistic” formula is model dependent. A different model would give a different translation between $\varepsilon$ and $\sigma$, and there exists no truly model-independent estimate of the EIS. In Section 4 we estimate $\sigma$ and $\varepsilon$ using the baseline model presented here, while in Section 5 we estimate these elasticities using our full stochastic lifecycle model.

### 3.3 Optimization Frictions

The model presented above assumes that there are no optimization frictions (such as inattention or misperception). However, some households may be prevented from bunching due to such frictions, in which case our estimate of $\sigma$ would be downward biased. To deal with this general
problem in empirical research, Kleven & Waseem (2013) developed a non-parametric frictions adjustment based on the presence of strictly dominated regions of behavior above notches. In their setting, strictly dominated regions above income tax notches were used to estimate the fraction of non-optimizing agents, while being agnostic about the specific reasons for not optimizing. Assuming that the fraction of non-optimizers is the same outside the dominated region (i.e., where it cannot be directly measured), Kleven & Waseem (2013) showed that it is possible to adjust the bunching estimates for the amount of optimization friction in order to estimate true structural elasticities.

Here we propose a parametric version of the Kleven-Waseem friction approach. In our setting, there are no strictly dominated regions per se. Locating just above an LTV notch implies a large drop in future consumption, but also allows for larger current consumption. If a consumer is perfectly impatient ($\delta = 0$), locating in such regions will be optimal. However, as long as consumers value future consumption at all ($\delta > 0$), there exists no non-negative elasticity of intertemporal substitution ($\sigma \geq 0$) that can justify locating immediately above an LTV threshold. Even with a zero substitution elasticity, the higher interest rate above the notch creates a wealth effect that should make consumers reduce consumption and leverage today, which is inconsistent with locating extremely close to the notch. Hence, we can structurally derive an LTV range above the notch that is inconsistent with any $\sigma \geq 0$. This range can be characterized as follows.

**Proposition 4.** For any $\sigma \geq 0$, choosing an LTV at the notch $\lambda^*$ dominates any interior LTV for households whose counterfactual LTV choice satisfies

$$\lambda^* + \Delta \lambda \in \left( \lambda^*, \left(1 + \frac{\Delta R}{R + 1}\right) \lambda^* \right).$$

(13)

**Proof.** The proof is in Appendix A.1.

We estimate the fraction of non-optimizers as the observed density mass in proportion to counterfactual density mass within the dominated region above the notch defined in (13). We assume that the fraction of non-optimizers in the dominated region is a good proxy for optimization frictions elsewhere in the distribution (where the amount of friction cannot be observed). Denoting the fraction of non-optimizers by $a$, the friction-adjusted bunching mass equals $\frac{B}{1 - a}$ and the frictionless LTV response equals $\frac{\Delta \lambda}{1 - a}$. These are the reduced-form statistics that enter into the structural estimation of $\sigma$.

Finally, it is worth noting that the dominated regions from which we estimate $a$ are very small.
As shown in Figure 1, the largest notch is at $\lambda^* = 80\%$ where the interest jumps by $\Delta R = 0.5\text{pp} = 0.005$. Using (13), this gives a dominated LTV range of approximately $(80\%, 80.2\%)$ for $R \approx 1$. For example, someone located at an LTV of 80.1% and with a house worth £200,000 (the average house value around this notch) would have to inject only £200 in order to get the 0.5pp reduction in the annual interest rate on the entire mortgage (worth roughly £3,000 in lower interest payments). If the household does not take this investment opportunity, we attribute it to optimization friction.

4 Estimating the EIS: Simple Approach

4.1 Bunching Estimation

In this section we use bunching to estimate the EIS and the reduced-form borrowing elasticity based on the simple framework developed above. The next section extends the analysis to our full structural model.

We first consider all notches together by pooling the data into a single average notch. For each notch point $n$ and each mortgage $i$, we calculate a normalized LTV as $LTV_{in} = LTV_i - n$.\(^{20}\) We then stack the normalized LTVs across the five notches and consider their distribution around the average notch at zero. This is shown in Figure 6 in which the actual LTV distribution is the series in black dots and the counterfactual LTV distribution is the series in orange crosses, with their frequencies in 0.25pp bins given on the left axis. The counterfactual is obtained using the panel method described in Section 2.4, with the global distribution shown in Figure 3. The figure also plots the pooled conditional interest rate at each LTV in green squares (right axis) obtained from the non-parametric regression described in Section 2.3.\(^{21}\) The graphs are shown with 95% confidence bands computed by bootstrapping.\(^{22}\)

The figure displays the two key empirical moments that we will use later: the interest rate jump at the notch, $\Delta r$, and the amount of bunching scaled by the counterfactual density at the threshold, $b \equiv B / [g_0 (\lambda^*) \times \text{binwidth}]$.\(^{23}\) As shown in equation (9), this bunching statistic is approximately

\[ b \approx B / [g_0 (\lambda^*) \times \text{binwidth}] \]

\(^{20}\)For example, a mortgage with an LTV of 73% has normalized LTVs of $LTV_{i70} = 3$ with respect to the 70% notch and $LTV_{i75} = -2$ with respect to the 75% notch.

\(^{21}\)The pooled interest rate line is a weighted average of conditional interest rates from the regression (1) implemented locally around each notch.

\(^{22}\)The bootstrap procedure draws 100 samples of mortgages from the observed sample, with replacement and strati-fying by notch.

\(^{23}\)We measure bunching $B$ as excess mass in the actual distribution relative to the counterfactual distribution in a 1pp range below the threshold, thus allowing for some overshooting by bunching households. Because bunching is extremely sharp in our setting, the results are very robust to reducing the width of this range.
proportional to the local average LTV response. In the figure, we distinguish between two different bunching statistics: \( b_{\text{raw}} \) and \( b \). The first estimate is based on the raw data shown in the figure, while the second (smaller) estimate adjusts for the presence of round-number bunching. As discussed earlier, we adjust for round-number bunching by using that some banks do not feature certain notches during certain periods.\(^{24}\)

The following main insights emerge from Figure 6. First, there is large and sharp bunching equal to five times the height of the counterfactual distribution at the notch, or about 10% less when accounting for round-number bunching. Second, there is a clear gap between the actual and counterfactual distributions to the right of the notch. This is the “hole” in which the bunching households would have been observed absent the notched interest rate schedule. The hole extends to around 3.5pp above the notch, implying that the most responsive households reduce their LTV by 3.5pp in response to the average interest rate notch of 0.25pp. The average LTV response (which relates to \( b \)) is of course smaller than the upper bound LTV response obtained from the end of the hole.

Third, there is a non-trivial number of mortgages observed just above the notch point. About 30% of borrowers stay in mortgages just above the notch rather than moving down to the notch. The standard interpretation of incomplete holes above notches is the presence of optimization frictions such as switching costs, inattention, and misperception (Kleven 2016). In this setting there is an additional reason for locating just above notches: the fact that some borrowers have mortgages in banks that do not feature that particular notch at the time of their loan origination. The impact of no-notch banks on the hole is limited by the fact that they represent a very small fraction of the data as mentioned above (see Figure A.5). Moreover, locating just above a threshold in a no-notch bank may in fact be viewed as an optimization friction in the form of bank switching costs. As Figure A.6 in appendix shows, borrowers located above a threshold in a no-notch banks could typically get a large discount by switching to a similar product in a different bank and moving below the threshold, and their unwillingness to do so must be related either to a friction in bank choice or to unobserved services provided by the no-notch bank. When we adjust for optimization

\(^{24}\)Figure A.5 in appendix shows the LTV distribution in the no-notch subsample, i.e. mortgages located close to one of the five thresholds in a bank-by-time cell that does not feature an interest notch at that threshold. Panel A of the figure shows that the no-notch subsample is a very small fraction of the full sample. Panel B of the figure zooms in on the no-notch subsample, demonstrating that there is a modest amount of round-number bunching. To adjust for round-number bunching, we estimate the degree of bunching at the notches using the no-notch subsample. As is common in the bunching literature, we estimate the counterfactual LTV distribution using a 5th order polynomial fitted to the data collapsed into bins with a width of 0.25 percentage points and excluding data within 1% of the notch. We then subtract the estimated normalized bunching in the no-notch sample, \( b_{RN} \), from the bunching moment of interest, \( b_{\text{raw}} \), to calculate our estimate of the bunching due to the interest rate notches.
frictions using the dominated region as shown in Section 3.3, we assume either that (i) all mass in
the dominated region is due to friction or (ii) that only the mass coming from notched banks is due
to friction. The latter assumption gives rise to a smaller friction adjustment and therefore a smaller
EIS, but we will show that our results are not very sensitive to this choice.

Figure 7 shows bunching evidence for individual notches, but is otherwise constructed in the
same way as the previous figure (we do not show the 60% notch, but it is qualitatively similar
to the ones displayed). The interest rate jumps shown in this figure are somewhat smaller than
those reported in Figure 1, because the interest rate jumps shown here apply to the population of
refinancers (as opposed to the full sample of buyers and refinancers) and therefore to a different
composition of mortgages. The evidence from individual notches is qualitatively consistent with
the evidence from the pooled notch. Excess bunching is in general large and leaves a hole in the
LTV distribution above each notch. The amount of bunching \( b \) is increasing in the size of the
interest rate jump, and the amount of mass just above the notch (friction) is decreasing in the size
of the jump, exactly as one would expect.

4.2 Elasticity Estimation

We turn to the estimation of the EIS in Table 2. The table shows results for the five individual
notches and for the pooled notch. Panel A starts by summarizing the statistics presented so far:
the interest rate jump \( \Delta r \) and the bunching statistic \( b \). The panel also shows the fraction of non-
optimizers \( a \) obtained from the dominated region, the friction-adjusted bunching statistic \( b_{Adj} = b / (1 - a) \), and the implied frictionless LTV response \( \Delta \lambda_{Adj} \). The average LTV response using the
pooled notch is close to 2pp, i.e. households are willing to reduce borrowing by an average of 2
percent of their house value in order to avoid the interest jump of 0.25pp. Apart from the 80%
notch, the LTV response is monotonically increasing in the size of the interest jump. At the 85%
notch where the interest rate jump is about 0.4pp, the LTV response is as high 3.7pp.

Are these reduced-form LTV responses large or small? Due to the discontinuous nature of the
interest incentive, there is no straightforward way of translating the LTV responses into mean-
ningful elasticities without specifying a model. The previous section showed the simplest possible
approach to make this translation. But before discussing elasticity estimates, Panel B presents a
set of intermediate reduced-form statistics that are helpful for interpreting the elasticities and for
understanding whether the effects should be viewed as large or small. This panel presents three
statistics: the amount of current consumption foregone due to LTV responses \( \Delta c_0 \), the amount of
future consumption gained due to LTV responses ($\Delta c_1$), and the implied annual rate of return on reducing LTV to the notch ($r^*$).

These statistics are calculated as follows. The reduction in current consumption is equal to $\Delta c_0 = \Delta \lambda \times P_0 H$, where $\Delta \lambda$ is the average LTV reduction shown in Panel A and $P_0 H$ is the average house value. Forgone current consumption equals £4,372 on average, and it ranges from £1,927 at the bottom notch to £6,425 at the top notch. The increase in future consumption $\Delta c_1$ includes three components: the additional housing equity from reducing the LTV to the notch $\Delta \lambda \times P_0 H$, the lower interest payments on inframarginal units of debt $(\Delta r \times \lambda^* \times P_0 H)$, and the lower interest payments on the marginal units of debt $((r + \Delta r) \times \Delta \lambda \times P_0 H)$. The returns from interest savings occur annually until the next refinancing event, $\Delta t$ years later. The increase in future consumption is very large, ranging from £2,600 to £9,521 across the different notches.

Using these estimates of $\Delta c_0$ and $\Delta c_1$, we calculate the (annualized) marginal rate of return on reducing the LTV to the notch as $r^* \equiv (\frac{\Delta c_1}{\Delta c_0})^{1/\Delta t} - 1$. This return equals about 10% on average and is as high as 13.83% at the 70% notch. This should be interpreted as a rate of return obtained by the marginal buncher who reduces LTV by $\Delta \lambda$, or alternatively the rate of return foregone by the marginal non-buncher at $\lambda^* + \Delta \lambda + \epsilon$. The fact that households are forgoing such large (and certain) returns raise a financial puzzle comparable to the widely discussed equity premium puzzle (e.g. Mehra & Prescott 1985; Kocherlakota 1996; Siegel & Thaler 1997). The marginal rates of return on bunching shown in Panel B are perhaps the most natural reduced-form way of assessing whether the LTV responses should be viewed as large or small. The large returns we estimate suggest that the responses are relatively small.

Finally, Panel C of the table turn our reduced-form evidence into elasticities using the simple structural model from Section 3. The EIS $\sigma$ is based on the estimating indifference equation shown in Proposition 1, and the reduced-form borrowing elasticity $\varepsilon$ is based on the equation on shown in Proposition 2. Besides the bunching moments, the estimation of $\sigma$ requires us to set a few additional parameters that are not directly observed. We make the following basic assumptions for this illustration of our method. We assume an (annualized) discount factor of $\delta = 0.96$, a typical value used in the macro literature. We assume that annual income is constant over time ($y_1 = y_0$), in which case the value of $\frac{y_1}{P_1 H} = \frac{y_1}{P_0 H}$ is observed in the data. To set $\Pi_1 = (1 - d) \frac{P_1}{P_0}$, we take housing depreciation from the literature as $d = 0.025$ and real house price growth as its historical

\footnote{As described in Section 2.1, the time between refinance events $\Delta t$ is in general determined by the timing of the reset rate and equals 3.34 years on average.}
average of $\frac{P_1}{P_0} = 1.006$. The period length is set equal to 3.34 years, corresponding to the average time between refinance events in the data. Admittedly, the most crude assumption in this exercise is the two-period nature of the model, but we have repeated the exercise for a multi-period version of the model and the EIS estimates are virtually identical (the multi-period extension is presented in appendix A.2).\(^{26}\)

As shown in the table, the EIS is small. It ranges from 0.03 to 0.18 across the different notches, and the average elasticity obtained from the pooled notch is close to 0.1. The small values of the EIS are consistent with the large returns to bunching shown in the panel B. A low EIS is required to justify that households are willing to forgo a certain 10% annual return on bunching. Changing the (crude) calibration underlying the estimation has little impact on this conclusion. Finally, we translate the structural EIS estimates into reduced-form borrowing elasticities using Proposition 2. We obtain stable values of $\varepsilon$ across the different notches, all in the neighborhood of 0.5. As shown above, when $\sigma$ converges to zero, the value of $\varepsilon$ converges to 0.5 and represent a pure wealth effect. Given the low values of the EIS deriving from our structural model, most of the borrowing response to interest rates is due to the wealth effect.\(^{27}\)

The elasticity estimates in Table 2 are corrected for optimization frictions using the Kleven-Waseem adjustment factor $a$, representing the fraction of borrowers located in the dominated regions above notches who do not move. The calculation of the dominated regions was based on the average interest jumps at notches in an empirical setting that feature heterogeneity in the size of interest jumps across banks and products. In particular, because some banks do not feature certain notches during certain time periods, it is conceivable that the mass immediately above notches disproportionately comes from such no-notch contracts. In this case we would overstate the amount of friction, and therefore overstate the EIS.\(^{28}\) There are other reasons why the Kleven-Waseem friction adjustment might be biased (see Kleven 2016). To investigate the sensitivity to these issues, Table 3 puts bounds on the friction adjustment and the EIS by considering the following scenarios: (i) no friction adjustment (lower bound on the EIS); (ii) a friction adjustment based on the dominated region and notched banks only; (iii) a friction adjustment based on the dominated region

\(^{26}\)While the structural EIS is not sensitive to the number of periods, the reduced-form borrowing elasticity does change when changing the number of periods (all else being equal). Adding periods to the model tends to lower the reduced-form elasticity by lowering (spreading out) the wealth effect of the interest rate.

\(^{27}\)Our analysis in the previous section suggests that reduced form elasticities are a function of $\sigma$ and $LTW$ and there is therefore no direct mapping between reduced form elasticities $\varepsilon$ and the structural elasticity $\sigma$. $LTW$ varies substantially across notches in Table 2. Nevertheless, reduced form estimates are similar across notches because our estimates of $\sigma$ are close to zero at all notches.

\(^{28}\)However, the potential bias is limited by the fact that the fraction of no-notch contracts is very small as we saw in Figure A.5.
and all banks (our baseline estimates); (iv) a friction adjustment assuming that all mass in the hole is due to friction (upper bound on the EIS). The table shows that the EIS estimates are relatively tight. Even under the extreme assumption that all mass in the hole is due to friction, the average EIS is fairly modest at around 0.3.

To conclude, we have translated bunching at mortgage notches into structural EIS parameters through the lens of the simplest possible dynamic model. In this model, the only way households can reallocate consumption over time is through borrowing against their house. This implies that bunching at mortgage notches maps directly into intertemporal consumption reallocation and therefore reveal the EIS. By assuming that households can bunch only by reducing current consumption, as opposed to drawing down liquid assets, we are estimating an upper bound on the EIS. The presence of liquid assets gives households an extra margin of adjustment and will in general add to bunching, unless liquidity demand is completely inelastic to interest rates. That is, a given amount of observed bunching is consistent with a smaller consumption reallocation in the presence of liquidity/portfolio effects, and by implication a smaller value of the EIS. This makes our small EIS estimates very informative.

As implied by this discussion, exactly identifying the EIS requires a model of how liquidity demand responds to the interest rate. This calls for a model that allows for uncertainty and a precautionary savings motive, in which case the responsiveness of liquidity depends on the amount of uncertainty and risk preferences. In our baseline model with CRRA preferences, the coefficient of relative risk aversion is the inverse of the EIS, and so a low EIS will make households very risk averse and therefore make liquidity demand inelastic. To allow for more flexibility in liquidity responses, it is natural to separate the EIS and risk aversion preferences using Epstein-Zin preferences. In the following section, we develop a stochastic lifecycle model that allows for all of these features in order to demonstrate how the EIS relates to bunching in a richer and more realistic setting. However, for the high-level reasons discussed here, the liquidity channel does not give rise to larger estimates of the EIS, all else equal.

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29 Adding fixed (price inelastic) liquid wealth to our model would have no impact on our results, because in that case borrowers still bunch using only consumption.

30 When moving to the full structural model below, we generalize in more dimensions than the liquidity channel. Due to the combination of generalizations, it is possible for the full structural estimates to be larger than the baseline estimates presented above.
5 Estimating the EIS: Full Structural Approach

5.1 Theoretical Framework

We now turn to estimating the EIS in our full structural model. Households live for \( T \) periods (years) and choose consumption, housing, liquidity, mortgage debt, and bequests. There is uncertainty about future house prices and income. Interest rates vary over time, and households face spreads over a base rate that are a function of LTV like the notched mortgage interest schedule in the UK. Households may buy and sell housing, and they may hold an additional liquid asset. As noted above, liquidity demand may be confounded with consumption demand in driving bunching behavior. The presence of a liquid asset in the model allows for an assessment of the relative importance of these two channels in creating bunching.

**Setup:** Households have preferences over non-durable consumption \( c_t \) and housing services \( H_t \), defined recursively as

\[
V_t = \left( (c_t^\alpha H_t^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + \delta \left( E_t \{ V_{t+1}^{1-\gamma} \} \right)^{\frac{1}{\gamma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{14}
\]

where \( V_t \) is the net present value of utility at period \( t \), \( \sigma \) is the EIS, \( \gamma \) is the coefficient of relative risk aversion, and \( \alpha \) is the share of housing in overall consumption. These preferences follow Epstein & Zin (1989) and Weil (1990) in allowing for a distinction between risk aversion and the EIS. This is important as we want to ensure that our EIS estimates are not influenced by the dual role this parameter plays in standard CRRA preferences as discussed in the previous section. To simplify the exposition of the model, we restrict attention to the special case of \( \gamma = \frac{1}{\sigma} \) (as in CRRA), but we relax this assumption in the empirical estimation of the model. The unit elasticity of substitution between housing and non-housing consumption (Cobb-Douglas) is justified by the observation that expenditures on housing services have historically been a constant share of total household expenditures (see Piazzesi & Schneider 2016 for a discussion).

Households value end-of-life wealth \( W_{T+1} \) via a reduced-form bequest motive, i.e.

\[
V_{T+1} = \Gamma W_{T+1}, \tag{15}
\]

where \( \Gamma \) is a parameter governing the intensity of bequest motives.

Households enter a period \( t \) with two assets and one liability. The assets are housing \( H_t \) with a
market price of $P_t$, and a liquid asset $L_t$ denominated in units of the numeraire consumption good. The liability is mortgage debt $D_t$, again denominated in units of the consumption good. The real gross mortgage interest rate is given by $R_t = 1 + r_t$, while the liquid asset obtains zero nominal return. For simplicity, we abstract from short-term credit so that households’ liquidity constraint is given by $L_t \geq 0$.

Mortgage contracts have fixed maturities of $m$ years, after which a penalizing reset interest rate kicks in. We assume this reset rate is sufficiently high that households never refinance after this point. Existing mortgages have a single exogenous amortization schedule with a share $\mu_t$ of outstanding debt due for repayment in period $t$. The typical mortgage contract in the UK requires full repayment before the borrower reaches the age of 70. Accordingly, we assume that all mortgage debt is repaid by period $T = 70$. Setting the model’s terminal period to this age understates average longevity in the UK, but households are typically inactive in the mortgage market after this point. One can think of the bequest function $V_{T+1}$ as capturing the overall preference for wealth at age 70 combining retirement and bequest motives.

When a household refinances, it must pay a fixed mortgage origination fee of $\Omega$ consumption units. If it refinances before the end of the fixed maturity $m$, there is an additional early repayment fee equal to a share $\omega$ of the outstanding loan. Households may move at any time, incurring a moving cost equal to a share $\theta$ of the value of the house, and they must refinance when moving. Houses depreciate in quality at a rate $d$ in each period. For simplicity, we assume that households maintain their houses in each period so as to replace its depreciated value, i.e. pay a maintenance fee of $d \cdot P_t H_t$ in period $t$. Households obtain an income stream of $y_t$ consumption units in period $t$.

With these features, the household’s budget constraint is given by

$$
c_t = y_t + (1 - \pi_t) L_t - L_{t+1} + P_t ((1 - d) H_t - H_{t+1}) + D_{t+1} - R_tD_t - \left( \Omega + \omega D_t I_t^R \right) I_t^R - \theta P_t H_t I_t^M,
$$

(16)

where $I_t^R$ is an indicator taking the value 1 if the household refinances ($D_{t+1} \neq (1 - \mu_t) D_t$) and 0 otherwise; $I_t^F$ is an indicator taking the value 1 if the household refinances early and 0 otherwise; $I_t^M$ is an indicator taking the value 1 if the household moves ($H_{t+1} \neq H_t$) and 0 otherwise; and $\pi_t$...
gives the inflation rate for non-housing consumption.

The interest rate is a spread over the risk-free rate, where the spread is a function of LTV corresponding to the notched mortgage interest schedule shown in Figure 1. Formally, \( R_t = R_t^0 + \rho(\lambda_s) \), where \( R_t^0 \) is the real risk-free rate and the spread \( \rho(\cdot) \) is a (step) function of the LTV ratio \( \lambda_s \) at the time \( s \) the mortgage was originated. The LTV ratio is given by \( \lambda_s \equiv \frac{D_{s+1}}{P_s H_{s+1}} \).

Agents in our model have rational expectations, are forward looking, and optimize in each period, subject to the aforementioned adjustment costs. This may seem at odds with our assessment that a large fraction of households face optimization frictions (reflected in our estimates of \( a \) in Table 2). However, in our empirical estimation of the model, we will use bunching moments adjusted for optimization frictions, as outlined in Section 3. We can therefore restrict attention to fully optimizing agents, as these moments reflect the amount of bunching that would prevail if households faced no optimization frictions. This has the advantage that we don’t have to take a position on the exact form of optimization frictions facing households in our theoretical framework.

In each period, households face a discrete choice among three options: they may remain in their existing house without refinancing, they may refinance their existing house, or they may move and obtain a new mortgage. We analyze these three options in turn.

**Liquidity Choice:** A household that neither moves \((H_{t+1} = H_t)\) nor refinances \((D_{t+1} = (1 - \mu_t) D_t)\) chooses consumption \(c_t\) and liquidity \(L_{t+1}\) to maximize lifetime utility, i.e.

\[
V^L_t(L_t, H_t, D_t) = \max\left\{ \frac{\sigma}{\sigma - 1} \left( c_t^{\gamma} H_{t+1}^{1-\gamma} \right)^{\frac{\sigma - 1}{\sigma}} + \delta E_t \{ V_{t+1}(L_{t+1}, H_{t+1}, D_{t+1}) \} \right\}
\]

subject to the budget constraint

\[
c_t = y_t + (1 - \pi_t) L_t - L_{t+1} - (r_t + \mu_t) D_t - dP_t H_t.
\]

\(V^L_t(\cdot)\) gives the value to a borrower entering period \(t\), if she chooses to remain in the same house and with the same mortgage. \(V_{t+1}(\cdot)\) gives the value to a borrower entering period \(t + 1\). To simplify the exposition, the utility function specified here corresponds to the special case of (14) where \(\gamma = \frac{1}{\sigma}\), but we will relax this assumption in the estimation. This maximization problem gives the following short-run Euler equation:

\[
\psi_t = \delta E_t \{ (1 - \pi_{t+1}) \psi_{t+1} \} + \zeta_t,
\]  

(17)
where $\zeta_t$ is the shadow value of relaxing the liquidity constraint, and $\psi_t$ is the marginal utility of non-durable consumption given by

$$
\psi_t \equiv \alpha \left( \frac{H_{t+1}}{c_t} \right)^{1-\alpha} \left( c_t^{\alpha} H_{t+1}^{1-\alpha} \right)^{-\frac{1}{\sigma}} .
$$

(18)

Equation (17) is a standard Euler equation that governs how a household draws down or accumulates liquidity in order to smooth non-housing consumption. The non-negativity constraint on liquidity creates a precautionary motive to hold liquid assets. In effect, a household that neither moves nor refinances faces a cake-eating problem as it runs-down liquidity until the next time it refinances.

**Mortgage Debt Choice:** A household that chooses to refinance its existing house faces the following decision problem

$$
V_t^R (L_t, H_t, D_t) = \max_{L_{t+1}, H_{t+1}, D_{t+1}, c_t} \left\{ \frac{\sigma}{\sigma - 1} \left( c_t^{\alpha} H_{t+1}^{1-\alpha} \right)^{\sigma-1} + \delta E_t \left\{ V_{t+1} (L_{t+1}, H_{t+1}, D_{t+1}) \right\} \right\}
$$

(19)

subject to the budget constraint

$$
c_t = y_t + (1 - \pi_t) L_t - L_{t+1} - dP_t H_t + D_{t+1} - \left( R_t + \omega I_t^E \right) D_t - \Omega .
$$

(20)

Here $V_t^R (\cdot)$ is the value to a borrower entering period $t$, conditional on refinancing. Recall that the interest rate in the following period(s) is a function of the choice of current LTV and therefore of current debt. Specifically, it is a flat function of LTV between notches and features discrete jumps at notches. Hence, the continuation value $V_{t+1} (L_{t+1}, H_{t+1}, D_{t+1})$ is discontinuous at the critical LTV ratios and therefore at critical values of debt $D_{t+1}$.\(^{31}\) The choice of debt $D_{t+1}$ can therefore be separated into a discrete and continuous component. We define $V_t^I (L_t, H_t, D_t)$ as the value of choosing the best interior value of debt, i.e. the value of maximizing (19) s.t. (20) while ignoring the presence of notches. Moreover, we define $V_t^N (L_t, H_t, D_t)$ as the value of borrowing to the notch, i.e. the value of maximizing (19) s.t. (20) when restricting to $D_{t+1} = \lambda^* P_t H_{t+1}$. A household chooses to bunch at the notch iff $V_t^N (L_t, H_t, D_t) \geq V^I (L_t, H_t, D_t)$. This is equivalent to the bunching decision in the 2-period model of Section 3 that led to the indifference equation (8). Hence $V_t^R (L_t, H_t, D_t) \equiv \max \left\{ V_t^N (L_t, H_t, D_t), V^I (L_t, H_t, D_t) \right\}$ gives the value of refinancing.\(^{32}\)

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\(^{31}\)This was also the case in the liquidity choice problem discussed above, but didn’t affect the analysis of liquidity choice.

\(^{32}\)The household may also choose to jump several notches, so formally this comparison must be done against all
Whether borrowing at the interior optimum or at the notch, liquidity choice is given again by (17). When refinancing, a household chooses the liquid buffer stock it wishes to store in anticipation of the cake-eating problem it will face while locked in to the current mortgage. The interior choice of debt is given by

$$\psi_t = -\delta E_t \left\{ \frac{\partial V_{t+1}}{\partial H_{t+1}} \right\},$$

where $H_{t+1} = H_t$ (not moving). The envelope theorem cannot generally be used to evaluate the marginal cost of debt (the right hand side of the equation), because of the fixed cost to refinancing and the discontinuities in the value function due to the notched mortgage schedule. But conceptually, the marginal cost of debt is driven by the discounted marginal utility of non-durable consumption at the next refinancing event. Specifically, if the time of next refinancing were known with certainty and the household never ran out of liquidity between mortgages, the first order condition would be rewritten as

$$\psi_t = \delta^{m} E_t \left\{ R_{t,t+m} \psi_{t+m} \right\},$$

(21)

where $R_{t,t+m}$ is the cumulative marginal cost of a unit of debt carried until the next refinancing year.\(^\text{33}\) This is a long-run Euler equation governing the choice of debt over the lifecycle. The long- and short-run Euler equations echo those studied in Kaplan & Violante (2014). Using their terminology, households in this model are wealthy hand-to-mouth: They have positive net worth, but can liquidate their wealth between refinancing episodes only at a cost. When they do not refinance, households are quasi-hand-to-mouth, as only their liquid wealth can be used for intertemporal substitution. In contrast, in a refinancing period, housing wealth becomes liquid again. Hence two separate Euler equations govern household behavior in these two instances. The short-term Euler equation governs the household’s liquidity management between mortgages and—when the liquidity constraint binds—their quasi-hand-to-mouth behavior. The long-run Euler equation determines the household’s longer-term lifecycle debt management choices.

How do the two Euler equations relate to each other? Assuming zero consumer good inflation

\(^{33}\) $R_{t,t+m}$ is a function of the mortgage interest rate, the inflation rate and the amortization rate in the years of the existing mortgage’s duration.
(to sharpen the intuition) and using the law of iterated expectations, the two combine to give

\[
E_t \left\{ \sum_{s=0}^{m} \delta^s \zeta_{t+s} \right\} = E_t \{ R_{t,t+m} \psi_{t+m} \} - E_t \{ \psi_{t+m} \}.
\]  

(22)

This equation equates the marginal benefit of paying down debt to that of holding liquidity. The left hand side of the equation gives the marginal value of holding liquidity, given by the expected net present value of the shadow cost of the liquidity constraint. The right hand side gives the marginal benefit of paying down debt. It gives the excess return on (paying down) mortgage debt relative to the (zero) return on liquid assets: The liquidity premium.

**Housing Choice:** A household that moves faces the following decision:

\[
V_t^M (L_t, H_t, D_t) = \max_{L_{t+1}, H_{t+1}, D_{t+1}, c_{t+1}} \frac{\sigma}{\sigma - 1} \left( c_t^0 H_{t+1}^{1-\alpha} \right)^{\frac{\alpha}{\sigma - 1}} + \delta E_t \{ V_{t+1} (L_{t+1}, H_{t+1}, D_{t+1}) \}
\]

subject to

\[
c_t = y_t + (1 - \pi_t) L_t - L_{t+1}
+ P_t ((1 - d - \theta) H_t - H_{t+1})
+ D_{t+1} - \left( R_t + \omega I^E_t \right) D_t - \Omega.
\]

The first-order conditions (17) and (21) still hold: The household is on its short-run and long-run Euler equations. In choosing a new mortgage, households face a similar bunching decision as in the refinancing decision described above. Housing choice is given by

\[
\left[ 1 - \delta (1 - d - \theta) E_t \left\{ \frac{P_{t+1} \psi_{t+1}}{P_t \psi_t} \right\} \right] \frac{P_{t+1} H_{t+1}}{c_t} = \frac{\alpha}{1 - \alpha}.
\]  

(23)

This first-order condition gives the relative expenditure on consumption \( c_t \) and housing \( H_{t+1} \). With Cobb-Douglas preferences, relative expenditure on commodities is equal to the ratio of their loadings in the Cobb-Douglass function (in this case \( \alpha \) and \( 1 - \alpha \)). However, in evaluating housing expenditure, the price of housing isn’t evaluated at its spot price \( P_t \), but also includes an additional term (given in in square brackets) that considers the asset value of housing.

**Refinancing and Moving Choice:** The household moves if \( V_t^M (L_t, H_t, D_t) \) exceeds both \( V_t^R (L_t, H_t, D_t) \) and \( V_t^L (L_t, H_t, D_t) \), it refinances without moving if \( V_t^R (L_t, H_t, D_t) \) is the largest of the three, and it remains in its existing house and mortgage contract if \( V_t^L (L_t, H_t, D_t) \) is the
largest of the three. Conceptually, households will choose to move when housing expenditure is sufficiently far from optimal, as per (23), so as to justify the moving cost $\theta$. Households refinance when they are sufficiently far off of their long-run Euler equations so as to justify the refinancing costs $\Omega$ and $\omega$. This occurs when interest rates are low relative to the value of liquidity (equity extraction decision) or interest rates are high and the household has sufficient liquidity (equity injection). However, as we have noted, mortgage contracts in the UK are typically short-term and with large pre-payment penalties $\omega$. Hence, in the UK, refinancing typically occurs at pre-determined dates (see Figure A.1). Our quantitative analysis will accord with this empirical regularity.

**Bequests:** Finally, in period $T$, the households may no longer borrow and choose housing and liquidity as follows:

$$V_T (L_T, H_T, D_T) = \max_{L_{T+1}, H_{T+1}} \frac{\sigma}{\sigma - 1} \left( (c_T^{\alpha} H_T^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + \delta (\Gamma V_{T+1})^{\frac{\sigma-1}{\sigma}} \right).$$

The overall magnitude of bequests is largely driven by the bequest parameter $\Gamma$. We evaluate terminal wealth at period $T$ prices. Hence there is no reason to bequeath any amount of the liquid asset unless house prices are expected to decline. Evaluating bequests at expected prices adds a portfolio motivation to bequeath some quantity of the liquid asset as a hedge against declining house prices, but doesn’t impact estimates of the EIS that are based on bunching decisions taken more than 30 years earlier.

### 5.2 Bunching and Solving for the EIS computationally

We now consider the bunching decision in more detail and how we confront it with the bunching moments to estimate the EIS. The model is solved computationally via backward induction starting from age 70 (bequest decision) and solving back to the age $\tau$ at which we observe households in the data (age 38 on average in the full sample, but this varies across cuts of the data). For each guess of $\sigma$, we iterate on the model to solve for the value function $V_{\tau+1} (L_{\tau+1}, H_{\tau+1}, D_{\tau+1}|\sigma)$. We use this value function to evaluate households’ continuation value as they make their refinancing choice. Households observed in the data are non-moving refinancers. In our model, they therefore face a choice of debt, liquidity, and consumption at time $\tau$. Given their debt choice $D_{\tau+1}$ and using initial wealth $W_\tau$, we can solve for optimal consumption and liquidity as the maximands of

$$V_\tau (W_\tau|\sigma) = \frac{\sigma}{\sigma - 1} \left( c_\tau^{\alpha} H_{\tau+1}^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} + \delta V_{\tau+1} (L_{\tau+1}, H_{\tau+1}, D_{\tau+1}|\sigma),$$
subject to the budget constraint

\[ c_\tau = W_\tau - L_{\tau+1} - (1 - \lambda_{\tau+1}) P_\tau H_{\tau+1}, \]  

(24)

where \( \lambda_{\tau+1} = \lambda^* \) when bunching and \( \lambda_{\tau+1} = \lambda^I \) is solved as the optimal interior LTV choice. In either case, debt is given by \( D_{\tau+1} = \lambda_{\tau+1} P_\tau H_{\tau+1} \). The solution of the liquidity-choice problem for the two cases gives value functions \( V_N^\tau (W_\tau | \sigma) \) (bunching) and \( V_I^\tau (W_\tau | \sigma) \) (interior). The marginal buncher is indifferent between bunching at locating at the optimal interior LTV. For this borrower, the indifference equation

\[ V_B^\tau (W_\tau | \sigma) = V_I^\tau (W_\tau | \sigma) \]  

(25)

holds and can be solved for \( \sigma \). This is done by repeating the entire process for a range of \( \sigma \) values and searching for the EIS that solves the indifference equation.

Of course, (25) contains parameters other than \( \sigma \) and a number of state variables. How, then, is \( \sigma \) identified from this equation? The discount factor \( \delta \) is an important determinant of the level of borrowing, but has only second order implications for the marginal response to interest rates, as discussed in Section 3. Accordingly, we find that our results are robust to a wide range of \( \delta \) and to hyperbolic discounting. Risk aversion \( \gamma \) could potentially play a role in bunching responses as it governs the elasticity of demand for liquidity. We experiment with a wide range of values for this parameter and show that for any degree of risk aversion, a low EIS is nevertheless necessary to explain the magnitude of bunching moments. Expectations are affected by the stochastic processes of house prices and income and the future path of interest rates (as well as the depreciation rate and bequest motives). We discuss their calibration below. However, as we show in our robustness analysis, our empirical methodology isn’t sensitive to the calibration of these processes. This is because expectations shift both sides of (25) by similar amounts and roughly cancel out from the estimating equation.

Finally, we do not observe initial wealth directly in our data, but use the method outlined in Section 3 to estimate it. That is, we back out initial wealth \( W_\tau \) from the optimality condition of the marginal buncher at the counterfactual. In the context of the full model, we define initial wealth as the sum of housing net worth and the liquid asset, net of the refinancing fee:

\[ W_\tau \equiv (1 - \pi_\tau) L_\tau + (1 - d) P_\tau H_\tau - R_\tau D_\tau - \Omega. \]

33
In the extended model studied here, a closed-form solution for initial wealth is unavailable, but we can solve computationally for initial wealth with the following steps.

1. Invert the Euler equations (17) and (21) and use the counterfactual LTV $\lambda + \Delta \lambda$ from the bunching moment to back out optimal consumption $c_\tau$ and liquidity $L_\tau + 1$.

2. Use the budget constraint (24), the counterfactual LTV, $c_\tau$, and $L_\tau + 1$ to back out initial wealth $W_\tau$.

To see how this is applied in practice, let $\lambda^* + \Delta \lambda$ be the counterfactual LTV estimated for the average marginal buncher. We observe house value $P_\tau \ H_{\tau + 1}$ in the data and can translate this into debt $D_{\tau + 1} = (\lambda^* + \Delta \lambda) \ P_\tau \ H_{\tau + 1}$. The solution of the lifecycle model gives us the value function $V_{\tau + 1} (\ L_{\tau + 1}, H_{\tau + 1}, \ D_{\tau + 1} | \sigma)$ and the long- and short-run Euler equations give

$$\psi_\tau = -\delta E_{\tau} \left\{ \frac{\partial}{\partial D_{\tau + 1}} V_{\tau + 1} (\ L_{\tau + 1}, H_{\tau + 1}, \ D_{\tau + 1} | \sigma) \right\}$$

and

$$\psi_\tau = \delta E_{\tau} \left\{ \frac{\partial}{\partial L_{\tau + 1}} V_{\tau + 1} (\ L_{\tau + 1}, H_{\tau + 1}, \ D_{\tau + 1} | \sigma) \right\}.$$  

The marginal utility of consumption $\psi_\tau$ is a function of consumption $c_\tau$ and housing $H_{\tau + 1}$ as in (18). Given housing $H_{\tau + 1}$, the two Euler equations can be solved (computationally) for consumption $c_\tau$ and liquidity choice $L_{\tau + 1}$. We can then use the budget constraint to back out initial wealth:

$$W_\tau = c_\tau - y_\tau + L_{\tau + 1} - (1 - (\lambda^* + \Delta \lambda)) \ P_{\tau + 1} \ H_{\tau + 1}. \ \ (26)$$

Initial wealth $W_\tau$ can then be applied to the budget constraint (24) when evaluating the indifference equation (25).

5.3 Calibration

Calibrated parameter values are summarized in Table 4. We now detail how these parameters were calibrated.

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34We observe the nominal value of housing $P_\tau \ H_{\tau + 1}$, but housing quality $H_{\tau + 1}$ is unobservable. In our baseline estimates, we assume households have the lowest house quality at the bunching choice, consistent with the lifecycle pattern of housing choices. Results were robust to allowing any value of initial housing quality. This is partially due to the unit elasticity between housing and non-housing consumption in our assumed preferences. Strong complementarities between housing and non-housing consumption would lead to behavior that is observationally equivalent to a low EIS in our model. See Flavin (2012) for a discussion. As we discuss below, strong complementarity between housing and consumption are a potential alternative explanation for the low EIS estimated in our model.
**General Assumptions:** The model is costly in terms of computational power. The bunching decision is central to our analysis. The problem therefore requires a fine grid for LTV choice between notches so as to ensure that we are evaluating the relative value of bunching and interior LTV choices correctly. We therefore make a number of simplifying assumptions to ease computation by reducing the state space on other dimensions. First, penalties of the magnitude of $\omega = 10\%$ of the outstanding loan are not uncommon in the UK. In initial simulations with fees of these magnitudes, households never found it optimal to refinance early. This is consistent with what we see in the data, where early refinancing is rare (Figure A.1 in the appendix). To simplify computation, we therefore assume that households always refinance when the reset rate kicks in and set $m = 3$, based on the average time to refinance in our data. The household faces a liquidity choice in all periods, as summarized by the short term Euler equation (17). In addition, the household faces a refinancing (and potentially housing) choice every third period. These variables are chosen in accordance with the the short term and long term Euler equations (17) and (21) and housing choice (23). We set the fixed refinancing cost to $\Omega = £1,000$, which is the origination fee on the typical mortgage product in the UK.

We also assumed a coarse grid of housing options, to aide computation, with three possible qualities, normalized to $\{1, 1.2, 1.4\}$. This distribution was sufficient to give the qualitative lifecycle of housing quality with increases in housing quality in midlife and decreases at end of life. With this discrete grid, the moving cost became not much more than a resource cost on households. In an initial round of simulations, it occasionally delayed moving by a year or two, but didn’t change the qualitative lifecycle housing pattern. We therefore abstracted from the moving cost (set $\theta = 0$).

**House Prices:** We assume house prices follow a log linear AR(1) process around a deterministic growth rate. Accordingly:

$$\ln P_t = p_0 + p_1 t + \rho_h \ln P_{t-1} + \varepsilon_t^P$$

$$\varepsilon_t^P \sim N(0, \sigma^2_p)$$

Using data from the mortgage lender Nationwide from 1974 to 2016 we calibrated the parameters of this process to $\rho_h = 0.875$, $p_1 = 0.006$, and $\sigma^2_p = 0.006$. We set $p_0$ so as to match the house price at the time of refinancing in our own data, i.e. we treat individual house prices as having a constant level shift relative to the national house price process. We will show that our results are robust to different assumptions about house-price growth and uncertainty.

**Income:** We assume that households face i.i.d. unemployment shocks around a deterministic age.
profile $y_t^{LC}$. The i.i.d assumption reduces the state space and eases computation. We will show that our results are robust to different degrees of income uncertainty and different lifecycle income patterns. Using HMRC data, the average lifecycle profile $y_t^{LC}$ is roughly quadratic with

$$y_t^{LC} = 1,360 \times \text{Age} - 14 \times \text{Age}^2 - y_0^i.$$  \hspace{1cm} (27)

In the data the average intercept is $y_0^i = 6,830$. However, we observe households’ income and age at time $t = \tau$: The bunching decision. We can therefore match individual’s $y_0^i$ based on their age in the data. In other words, we treat the household’s cross-sectional deviation from the average age-income profile as a permanent level shift.

We set the probability of unemployment to 5%, roughly the historical average, although results are robust to different probabilities as we show in our robustness analysis. Applying formulae for unemployment benefits to the typical household in our sample gave a replacement rate of approximately 60% in the first year of unemployment when considering all available benefits, including the universal credit and the job seeker’s allowance. Given our i.i.d. assumption, households rarely face an unemployment spell exceeding a year.\footnote{One might expect the probability of unemployment to be lower for homeowners than the general population. Moreover, couples comprise half our sample and their replacement rate is higher if only one breadwinner is unemployed. Our results are robust to a wide range of unemployment probabilities and replacement rates. Generally, unemployment affects liquidity choice, but not the estimated EIS.}

**INTEREST RATES:** We assume households face a fixed interest rate for the $m = 3$ years of the mortgage. Mortgage interest rates have a risk premium $\rho(\lambda)$ over the Bank of England Policy (real) rate $r_0^t$. We assume that the risk premium is a constant function of LTV as represented in the notched LTV schedule shown in Figure 1, but that the reference policy rate varies over time. We assume that the policy rate follows a deterministic time path to reduce the dimensionality of the problem and ease computation. We forecast the (real) Bank of England policy rate with forward rates implied by the UK yield curve. This implies a slowly increasing path of interest rates over time.

**INFLATION (EXPECTATIONS):** We assume inflation is 2% a year each year, as per the Bank of England’s target. Higher or stochastic inflation has some implications for portfolio choice (high inflation expectations make nominal liquid assets relatively less attractive), but little implication for the estimated EIS.

**BEQUEST MOTIVE:** We experimented with a range of parameters for the bequest motive $\Gamma$. Be-
quests are 30 years removed from the bunching decision for the average household in our sample and thus have little impact on our estimates of the EIS. The median British household leaves no bequests and the median British homeowner leaves only housing wealth as a bequest. The assumption $\Gamma = 0.1$ leads to bequests that are of similar magnitude to those observed in the data and we use this in our baseline simulations.

**Amortization:** We assume a simple amortization schedule with constant annual repayments ensuring full repayment by age 70. That is, the amortization rate is given by

$$\mu_t = \frac{1}{70 - \text{Age} + 1}.$$  

Of course, households have ample opportunities to readjust amortization by repaying or extracting equity when refinancing.

**Risk Aversion:** We estimate the model with Epstein-Zin-Weil preferences. In our baseline estimates, we set risk aversion to $\gamma = 2$, as is common in the macro literature. We conduct robustness analysis with respect to risk aversion, including the possibility of $\gamma = \frac{1}{\sigma}$, e.g. CRRA preferences.

**Depreciation:** Harding et al. (2007) estimate an annual depreciation rate of $d = 0.025$ per annum. This rate is close to UK estimates of the office of the deputy prime minister, as reported by Attanasio et al. (2012).

**Discounting:** We set $\delta = 0.96$, as is common in the literature and conduct robustness checks with respect to this parameter. We also allow for hyperbolic discounting in additional robustness checks.

### 5.4 Model Simulation

As we turn to the results, it is useful first to illustrate the workings of the extended model. We do so by showing the output from a single (illustrative) simulation round of the model. The output is shown in Figure A.7. This simulation assumes $\sigma = 0.1$, which is within our range of estimates. We consider a household that begins in the neighborhood of the 75% interest notch at age 38 (a typical household in our sample). The top panels show the evolution of the exogenous state variables: house prices and income. House prices grow over time, but feature two slumps in this simulation round: one beginning when the borrower is 41 years old and the other in her late 50s. Income follows a lifecycle pattern, peaking in the early 50s. This pattern is punctuated by a brief spell of unemployment at age 49.
The middle panels show the borrower’s housing and mortgage choices. Housing follows a lifecycle pattern: the borrower begins with the lowest housing quality (by assumption) and increases her house quality in her early 40s and again in her late 40s. She then downsizes late in life. Note that this occurs without any exogenously imposed lifecycle needs (e.g., children), but rather through an endogenous accumulation of wealth over the lifecycle. The borrower times her housing upgrades to take advantage of low prices during the first housing slump.

Turning to leverage, the borrower repays her mortgage over the lifecycle at the assumed amortization rate between mortgages. Early in life, the borrower tends to extract equity when refinancing and she then repays more towards the end of life. The household borrows to finance the two housing upgrades and notice that that household bunches at the 70% notch at the second move.

Turning to the bottom panels, the household chooses its lifetime amortization profile to smooth consumption over the lifecycle, giving a flat lifetime consumption profile.\textsuperscript{36} There is a gradual increase in consumption at the end of the life, because the precautionary savings motive represses consumption at earlier ages. The net present value of lifetime (non-housing) consumption is smaller than the net present value of income because of housing purchases, maintenance, and debt servicing. House prices follow a very persistent process, so that the household views house price increases as reflecting substantial increases in net worth and she increases her consumption accordingly. This is particularly true late in life, when the increase in net worth allows a large increase in annual consumption. At each refinancing episode, the household takes out a substantial amount of equity in the liquid asset (around £30,000) and runs down this liquidity over the duration of the mortgage, as suggested by the short-term Euler equation.\textsuperscript{37} Recalling that the borrower faces an unemployment spell at age 49, note that she rapidly draws down her liquid holdings to smooth the shock. This shock unfortunately hits shortly after she upgraded her property. Hefty precommitted mortgage repayments combined with the income loss due to unemployment rapidly exhausts the borrower’s liquid holdings and requires a reduction in consumption. This is precisely the “wealthy hand-to-mouth” behavior that Kaplan & Violante (2014) postulate.

\textsuperscript{36}A hump-shaped consumption profile would be more empirically accurate and could be easily accommodated by adding an exogenous demand shifter. This addition has essentially no implications for the estimated EIS. We therefore abstract from lifecycle consumption preferences in the current specification.

\textsuperscript{37}The increased demand for liquid holdings later in life is because the liquid asset is the only financial asset available to households in the model and the household has already the maximal house quality allowed in our model.
5.5 Results

Table 5 shows the estimated EIS resulting from this extended model. Note that estimates are not based on the simulations shown in Figure A.7 or indeed any other simulation. Instead, they follow from the solution of the indifference equation (25), using the value functions arising from the model. Standard errors are obtained by block bootstrapping with replacement. The first row of the table restates our estimates of the amount of bunching \( b \), as reported in table 2. This is given at each notch and at an “average” notch, estimated as a weighted average of the bunching masses at individual notches, with weights proportional to the number of borrowers around each notch in the counterfactual distribution.\(^{38}\) The table also restates the fraction of borrowers \( a \) affected by optimization frictions, the bunching estimate adjusted for frictions \( b_{\text{Adj}} = \frac{b}{1-a} \), and the implied LTV response \( \Delta \lambda \) for optimizers.

Finally, in the last row of Table 5, we report our estimates of the EIS \( \sigma \) based on the solution to the full structural model. The EIS is small and stable across notches, with the exception of the 85% LTV notch where the elasticity is somewhat larger. The average EIS is slightly below 0.1. This average EIS is essentially identical to that deriving from the simple two-period model shown in Table 2, despite all the bells and whistles of the full structural model. This suggests that the low EIS we have obtained are not due to model specification, but rather due to the magnitude of bunching observed in the data.\(^{39}\)

Why does model specification matter so little? Despite the greater complexity of the model developed here compared to the two-period model in Section 3, solving for the EIS ultimately boils down to a similar indifference equation. There are two main differences between the models. First, the continuation value in the two-period model is simply the utility of second-period consumption, as opposed to the much more involved continuation value \( V_{\tau+1}(L_{\tau+1}, H_{\tau+1}, D_{\tau+1} | \sigma) \) in the full lifecycle model. This has little quantitative implication for EIS estimates. This is because most factors that affect the continuation value \( V_{\tau+1}(L_{\tau+1}, H_{\tau+1}, D_{\tau+1} | \sigma) \) affect it by similar magnitudes

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\(^{38}\)When using the full structural model, we cannot estimate the EIS at the pooled notch, as we did in Section 4, because the pooled notch would be located at an LTV of approximately 73% where no actual notch exists. This is tightly located between the existent 70% and 75% notches so that households would often jump more than one notch in the neighborhood of such a series of notches.

\(^{39}\)Why does the 85% notch yield a larger EIS? Our model implies substantial equity injection at the counterfactual at the time of bunching for the average household at this notch. This means that households are short of liquidity when attempting to bunch at the 85% notch and must forgo some current consumption for capital repayment even at the counterfactual. This makes bunching at the 85% notch particularly painful and justifies the small amount of bunching observed in the data even at a higher EIS approaching 0.3. This result illustrates how our model incorporates liquidity constraints in its estimation, but also that the EIS remains low even when incorporating severe liquidity constraints. We do not observe households’ wealth directly, but if households at this notch have more wealth than our model suggests, our estimated EIS is overstated.
when bunching at the notch and when locating in the interior, and therefore roughly cancel out from the two sides of the indifference equation (25). Rather, it is the curvature of the continuation value with respect to debt that governs bunching motivations and therefore EIS estimates. But the curvature of the value function \( V_{\tau+1}(\cdot) \) with respect to wealth (and therefore debt) is approximately equal to the EIS, as is often the case in dynamic consumption models. Hence the bunching decision is roughly the same in the full model as it was in the two-period model.

Second, our full structural model allows for a liquid asset and for liquidity choice, while no liquid asset was available in the two-period model. As noted earlier, this is a more substantive difference, because the liquid asset gives households an additional margin of adjustment when making the bunching decision. With liquid assets, households confronting the bunching decision can now forgo either consumption or liquidity to lower their LTV to the notch. In equilibrium, households equalize the marginal value of consumption with the marginal value of liquidity, so it is optimal to forgo a combination of the two, a point illustrated in equation (17). The moderate response to interest rates implied by the bunching moment suggests that households are inelastic in their demand for consumption and liquidity. With reasonable values of risk aversion and uncertainty — the main parameters governing liquidity demand — the liquidity margin facilitates bunching, which heightens the “puzzle” of households’ small responses to borrowing rates. In contrast, the simple model without liquidity is equivalent to a model with a binding liquidity constraint or one with perfectly inelastic liquidity demand. Hence, the model with liquidity should give smaller EIS estimates all else equal. However, because the EIS estimates from the baseline model were already small, incorporating liquidity choice has only a small impact on the estimation.

Having observed some heterogeneity in the EIS across LTV ratios, we turn to other dimensions of potential heterogeneity in Table 6. In this table we analyze how our EIS estimate varies with age, income, loan to income, income growth, house price growth, and the interest rate change since the previous mortgage (assuming passive borrower behavior). For each covariate, we estimate \( \sigma \) in four quartiles of the distribution of the covariate, averaging across the five notches.

The overall finding from Table 6 is that there are no dramatic patterns of heterogeneity. The EIS is relatively stable in most dimensions and our estimates are always below 0.2, a modest value compared to most previous estimates and to benchmark assumptions in calibration studies. Specifically, we find that the EIS is monotonically increasing in age, income, and the loan-to-income ratio, but only moderately so. Given that age, income, and loan to income are highly correlated with household wealth (which we do not observe in the data), these findings suggest that heterogeneity
with respect to wealth is not very strong either. Wealth and income have been argued to be important sources of EIS heterogeneity (see Guvenen 2006), but we do not find strong heterogeneity in these dimensions in our sample of homeowners with mortgages. However, this sample excludes the very wealthiest segments of the population (who typically do not have mortgages) where the EIS could conceivably be higher.

5.6 Robustness

We have shown that our EIS estimates are largely robust to first-order changes in modeling assumptions, i.e. moving from a very simplified 2-period model to a full dynamic lifecycle model. In this section we subject our model to an additional battery of tests to explore its robustness to specification and parameter values. Table 7 shows EIS estimates (at the average notch) across a range of parameterizations. In each row, the estimate in boldface corresponds to the baseline assumption considered above.

We start by exploring robustness to household impatience. The first row shows robustness to the discount factor $\delta$, while the second row allows for hyperbolic discounting and varies the coefficient of present bias $\beta$. For this latter exercise, we alter the model so that households maximize the following function in each period

$$
\left( \left( \alpha t H_{t+1}^{\frac{1-\alpha}{\sigma}} \right)^{\frac{\sigma-1}{\sigma}} + \beta \delta \left( E_t \left\{ V_{t+1}^{1-\gamma} \right\} \right)^{1-\gamma} \frac{\sigma-1}{\sigma} \right)^{1-\gamma},
$$

with $V_{t+1}$ defined as in the baseline model. As noted earlier, the discount factor (or present bias) largely governs the level of borrowing, while the EIS governs the responsiveness of borrowing to interest rates. As a result, differences in the discount factor $\delta$ or the present bias factor $\beta$ have relatively little impact on the estimated EIS. Even under extreme present bias ($\beta = 0.3$), the EIS estimate is only moderately larger than in the baseline.

We next turn to risk aversion, which governs liquidity preferences and may affect EIS estimates. We explore values of $\gamma$ ranging from risk neutrality ($\gamma = 1$) to high levels of risk aversion. The last column shows the CRRA case $\gamma = \frac{1}{\sigma}$, which implies extreme risk aversion due to the low range of $\sigma$ in our estimates. Despite the potential role of liquidity in affecting our estimates, the EIS remains relatively small for the entire range of risk preferences.

The remaining robustness checks alter household expectations along a number of dimensions. First, we alter interest rate expectations. The calibrated interest rate process was calibrated from
the UK yield curve, which currently reflects expectations of interest rates near zero for an extended period. We see that our estimated EIS remains similar if households instead expect higher interest rates (shifting the yield curve upwards by 1pp, 2pp, and 3pp). Second, we see that the estimated EIS is insensitive to expectations of (mean) house price growth and not particularly sensitive to uncertainty about house prices. We alter the expected lifecycle profile of income from static income expectations to a trajectory implying real income growth at 7% per annum. Again, the implications for the estimated EIS are small. Finally, we alter income uncertainty by either changing the probability of unemployment or altering the replacement rate for unemployed workers. In both cases, the EIS remains moderate.

The message emerging from our robustness checks is that our EIS estimates are insensitive to parametric assumptions in the model. The similarity between our estimates in the two period and the fully structural model lead us to believe that our results, and our conclusion of a low EIS, are robust to model specification.

Our robust results may raise the question of whether our EIS estimates reflect a true aspect of household behavior, or whether there is something peculiar about the bunching methodology or the class of models we have used that “necessitate” a low EIS. There are several indicators that our estimates are not driven by methodology or modeling choices. First, recall that the incentives to bunch are enormous. Our estimates in Table 2 indicate that the marginal non-bunching household is forgoing risk-free returns in excess of 10% by not moving down to the notch. This is a direct — model free — indication that households are unresponsive to large interest rate incentives. Our structural estimation can be viewed as translating this lack of responsiveness in an EIS parameter. Second, we have experimented with a variety of hypothetical bunching moments and were able to generate larger values of the EIS. For example, if the bunching observed in the data reflected responses to interest rate notches half the size actually observed, the indifference equation would yield EIS estimates exceeding 0.5. But given the magnitude of interest rate notches actually observed in the data, we would see far more bunching if the EIS was larger.

Finally, we note that the low elasticity of intertemporal substitution could be explained by other behavioral models and frictions than those considered here. For example, Campbell & Cochrane (1999) show that consumption habits may lead to low elasticities of intertemporal substitution. Chetty & Szeidl (2016) show that consumption commitments (captured by adjustment costs in consumption) may generate similar behaviors as consumption habits, and they therefore represent
another behavioral reason for low values of the EIS.\footnote{However, note that non-responses due to consumption commitments may be partly captured by the Kleven-Waseem friction adjustment approach.}

6 Conclusion

A large literature estimates the elasticity of intertemporal substitution using time series, panel data, and structural models. The EIS is arguably one of the most important parameters in economics as it plays a central role in almost any economic model involving intertemporal choice. It governs consumption and savings responses to interest rate changes, affects the reaction of consumption to income shocks, is an important parameter for asset pricing, and provides a key statistic for evaluating a range of macroeconomic and microeconomic policies. We contribute to efforts to estimate the EIS with a new approach that combines quasi-experimental identification with structural methods.

Our new methodology translates empirical moments arising from bunching at mortgage interest notches into EIS estimates. Using administrative mortgage data from the UK, we first illustrate our approach using a simple two-period model, and then generalize to a rich stochastic lifecycle model. The two models produce very similar results: the EIS is small, around 0.1-0.2. Although we observe lots of bunching at notches, the intertemporal incentives created by those notches are so large that there would have to be much more bunching to justify large values of the EIS. Our results are close to the level obtained in the early macro-based literature (e.g. Hall 1988), although we have arrived at this conclusion using a fundamentally different approach.

We also use our framework to clarify the relationship between the structural EIS and the reduced-form borrowing (or saving) elasticities often estimated in the literature. We show that, in general, reduced-form estimates are not particularly informative of deeper structural elasticities, because the relationship between the two is mediated by endogenous variables that may vary widely across settings and samples. We provide guidance on how to translate reduced-form estimates into estimates of the EIS through a simple sufficient statistics formula.
References


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Notes: The figure shows the conditional interest rate as a function of the Loan-To-Value (LTV) ratio from the non-parametric regression (1). In each LTV bin, we plot the coefficient on the LTV bin dummy plus a constant given by the mean predicted value $E[\hat{r}_i]$ from all the other covariates (i.e., omitting the contribution of the LTV bin dummies). The figure shows that the mortgage interest rate evolves as a step function with sharp notches at LTV ratios of 60%, 70%, 75%, 80%, and 85%.
Notes: The figure shows the distribution of Loan to Value (LTV) ratios of mortgages in the full sample of house buyers and refinancers (panel A) and in the panel of refinancers (panel B). Small dots denote the number of mortgages in bins 0.25pp wide, while larger dots denote the number of mortgages in bins whose upper bounds are located at the notch points (denoted by dashed, vertical lines) of 60%, 70%, 75%, 80%, 85%, 90%, and 95% LTV.
Notes: The figure shows the two steps in the construction of the counterfactual Loan to Value (LTV) distribution in the refinancer panel. Both panels show the actual LTV distribution in black dots, constructed in the same way as described in the note to Figure 2. Panel A shows the distribution of passive LTVs in orange crosses, based only on the LTV of the previous mortgage, amortization, and the house value at the time of refinancing. Panel B shows the distribution of counterfactual LTVs in orange crosses, which adjusts passive LTVs for the average equity extraction of non-bunchers in the actual distribution.
Figure 4: Bunching in a Simple Model of Intertemporal Substitution

Panel A: The Marginal Bunching Household

Panel B: LTV Density Distribution

Notes: The figure shows the choice faced by a refinancing household faced with a notched interest rate schedule in a budget set diagram with LTV $\lambda$, on the horizontal axis and consumption in the future, $c_1$, on the vertical axis. Given the period zero budget constraint and the value of housing, an LTV choice $\lambda$ translates into current consumption $c_0$. The solid black line depicts the budget constraint with an interest notch at the LTV $\lambda^*$. The budget constraint has a slope $-R$ below $\lambda^*$ and a slope $-(R + \Delta R)$ above $\lambda^*$. The indifference curves shown are those of the marginal bunching household that is indifferent between bunching at the notch and borrowing $\lambda^*$, and a point $\lambda'$ in the interior of the higher interest rate bracket. This household would have chosen a point $\lambda^* + \Delta \lambda$ in the absence of the interest rate notch. As described in Section 3, the optimality conditions for $\lambda'$ and $\lambda^* + \Delta \lambda$ together with the household’s indifference between $\lambda'$ and $\lambda^*$ identify the curvature of the indifference curves—the Elasticity of Intertemporal Substitution (EIS)—in terms of observable and/or estimable quantities.
Notes: The figure shows the reduced form elasticity of borrowing with respect to the interest rate $\varepsilon$ as a function of the structural elasticity of intertemporal substitution $\sigma$. The correspondence follows from the simple model in Section 3 and is given by (11). The three curves correspond to three values of the loan-to-wealth ($LTW$) ratio, which is the ratio of the mortgage loan to total future housing and human wealth. The reduced form elasticity is increasing in $\sigma$ but also affected by $LTW$. A given reduced form estimate is thus consistent with a wide range of structural estimates of the EIS.
Notes: The figure shows the actual ($f(\lambda)$ in black dots) and counterfactual ($f_0(\lambda)$ in orange crosses) distributions of LTV ($\lambda$), pooling all notches (60%, 70%, 75%, 80%, 85%). The green squares show the conditional interest rate in each LTV bin from the regression described in Section 2.3 and the footnote to Figure 1. The counterfactual LTV distribution is obtained using the method outlined in Section 2.4 and the footnote to Figure 3. The figure also shows the jump in the conditional interest rate at the notch $\Delta r$; the (raw) normalized amount of bunching in the actual distribution $b$(raw), calculated as described in Section 3; and $b$ which is the amount of bunching net of round number bunching.
Notes: The figure shows the actual \( f(\lambda) \) in black dots) and counterfactual \( f_0(\lambda) \) in orange crosses) distributions of LTV \( \lambda \), at the notches at 70% (Panel A), 75% (Panel B), 80% (Panel C) and 85% (Panel D) LTV. The green squares show the conditional interest rate in each LTV bin from regression described in Section 2.3 and the footnote to Figure 1. The counterfactual LTV distribution is obtained using the method outlined in Section 2.4 and the footnote to Figure 3. The figure also shows the jump in the conditional interest rate at the notch \( \Delta r \); the (raw) normalized amount of bunching in the actual distribution \( b(\text{raw}) \), calculated as described in Section 3; and \( b \) which is the amount of bunching net of round number bunching.
<table>
<thead>
<tr>
<th></th>
<th>(1) Full sample mean/sd</th>
<th>(2) Remortgagors mean/sd</th>
<th>(3) Remortgagor panel mean/sd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate (%)</td>
<td>4.12 (1.29)</td>
<td>4.26 (1.38)</td>
<td>4.06 (1.23)</td>
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<tr>
<td>Loan Size (£)</td>
<td>142,710.6 (120,512.8)</td>
<td>138,376.2 (120,725.2)</td>
<td>145,400.8 (116,489.4)</td>
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<tr>
<td>Property Value (£)</td>
<td>257,004.0 (255,284.2)</td>
<td>263,076.4 (270,122.1)</td>
<td>262,304.0 (247,074.1)</td>
</tr>
<tr>
<td>Loan to Value Ratio (%)</td>
<td>59.9 (22.1)</td>
<td>56.4 (21.7)</td>
<td>58.8 (19.8)</td>
</tr>
<tr>
<td>Gross Income (£)</td>
<td>57,270.6 (84,069.3)</td>
<td>57,480.9 (84,517.4)</td>
<td>57,483.2 (77,513.0)</td>
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<tr>
<td>Loan to Income Ratios</td>
<td>2.78 (1.85)</td>
<td>2.68 (2.03)</td>
<td>2.78 (1.60)</td>
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<tr>
<td>Repayments to Income (%)</td>
<td>21.8 (39.5)</td>
<td>22.6 (48.4)</td>
<td>21.9 (15.4)</td>
</tr>
<tr>
<td>Proportion with Joint Income</td>
<td>0.54 (0.50)</td>
<td>0.55 (0.50)</td>
<td>0.55 (0.50)</td>
</tr>
<tr>
<td>Mortgage Term (years)</td>
<td>20.7 (7.44)</td>
<td>19.0 (6.92)</td>
<td>20.0 (6.65)</td>
</tr>
<tr>
<td>Proportion of Fixed Rate Mortgages</td>
<td>0.68 (0.47)</td>
<td>0.65 (0.48)</td>
<td>0.73 (0.45)</td>
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<tr>
<td>Proportion of Remortgagors</td>
<td>0.56 (0.50)</td>
<td>0.88 (0.32)</td>
<td>0.76 (0.42)</td>
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<tr>
<td>Borrower’s Age</td>
<td>40.0 (10.0)</td>
<td>41.7 (9.57)</td>
<td>40.3 (8.93)</td>
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<td>Observations</td>
<td>3,049,164</td>
<td>1,961,325</td>
<td>647,192</td>
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</table>

Notes: The table shows summary statistics of the three samples used in our analysis. Column (1) shows our full sample: all usable mortgages in the PSD (house purchases and refis) including (a) observations where we can find information on the product fee in MoneyFacts (b) any refiner’s previous mortgage (which we can use without fee information). Column (2) restricts the sample to refiners only (all refinance products and any refiner’s previous mortgage). Finally, column (3) shows the sample of refiners in the panel (refiners we can link over time and where we can construct a counterfactual LTV ratio).
### Table 2: From Bunching to the EIS: Simple Model

<table>
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<tr>
<th>Statistic</th>
<th>60</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>Pooled</th>
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<tr>
<td><strong>Panel A: Bunching Evidence</strong></td>
<td></td>
<td></td>
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<tr>
<td>( r(%) )</td>
<td>3.17</td>
<td>3.25</td>
<td>3.44</td>
<td>3.76</td>
<td>4.38</td>
<td>3.42</td>
</tr>
<tr>
<td>(0.01)</td>
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<td></td>
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</tr>
<tr>
<td>( \Delta r(%) )</td>
<td>0.10</td>
<td>0.21</td>
<td>0.33</td>
<td>0.37</td>
<td>0.39</td>
<td>0.25</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>( b )</td>
<td>0.96</td>
<td>2.94</td>
<td>6.86</td>
<td>6.42</td>
<td>7.45</td>
<td>4.45</td>
</tr>
<tr>
<td>(0.17)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>( a )</td>
<td>0.58</td>
<td>0.21</td>
<td>0.30</td>
<td>0.15</td>
<td>0.08</td>
<td>0.29</td>
</tr>
<tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td>( b_{Adj} )</td>
<td>2.31</td>
<td>3.73</td>
<td>9.87</td>
<td>7.59</td>
<td>8.11</td>
<td>6.30</td>
</tr>
<tr>
<td>(0.49)</td>
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<td></td>
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<td>(0.01)</td>
</tr>
<tr>
<td>( \Delta \lambda_{Adj} )</td>
<td>0.67</td>
<td>1.06</td>
<td>3.32</td>
<td>2.68</td>
<td>3.71</td>
<td>1.93</td>
</tr>
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<td>(0.14)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>(0.01)</td>
</tr>
<tr>
<td><strong>Panel B: Implicit Interest Rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta c_0 )</td>
<td>1,927</td>
<td>2,607</td>
<td>7,288</td>
<td>5,230</td>
<td>6,425</td>
<td>4,372</td>
</tr>
<tr>
<td>(388)</td>
<td></td>
<td></td>
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<td></td>
<td>(210)</td>
</tr>
<tr>
<td>( \Delta c_1 )</td>
<td>2,600</td>
<td>3,965</td>
<td>9,521</td>
<td>7,778</td>
<td>9,362</td>
<td>6,040</td>
</tr>
<tr>
<td>(415)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(224)</td>
</tr>
<tr>
<td>( r^{*}(%) )</td>
<td>9.58</td>
<td>13.83</td>
<td>8.60</td>
<td>11.84</td>
<td>10.85</td>
<td>10.11</td>
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<td>(0.47)</td>
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<tr>
<td><strong>Panel C: Elasticities</strong></td>
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<td></td>
</tr>
<tr>
<td>EIS ( \sigma )</td>
<td>0.03</td>
<td>0.03</td>
<td>0.18</td>
<td>0.08</td>
<td>0.14</td>
<td>0.08</td>
</tr>
<tr>
<td>(0.01)</td>
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<td>(0.01)</td>
</tr>
<tr>
<td>Reduced-form ( \varepsilon )</td>
<td>0.53</td>
<td>0.53</td>
<td>0.60</td>
<td>0.56</td>
<td>0.58</td>
<td>0.55</td>
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<td>(0.01)</td>
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<td>(0.00)</td>
</tr>
</tbody>
</table>

Notes: The table shows our reduced form estimates using bunching at the various LTV notches separately, and pooling the notches from 60\% to 85\%. \( r \) is the conditional nominal interest rate below the notch, \( \Delta r \) is the interest rate jump at the notch, estimated as described in Section 2.3, \( b_{Adj} \) is our normalized bunching estimate, net of round number bunching and adjusted for optimization frictions, and \( \Delta LTV_{Adj} \) is our estimate of the leverage response, estimated as described in Section 3. \( \Delta c_0 \) is the average foregone current consumption by bunchers, and \( \Delta c_1 \) is the average increased future consumption for bunchers, implying an annualized rate of return of \( r^{*} = (\Delta c_1 / \Delta c_0)^{1/\Delta t} - 1 \), where \( \Delta t \) is the duration of the lower initial interest rate. The EIS \( \sigma \) is estimated using a simple 2-period model described in Section 3: the solution to the indifference equation (8). The reduced form elasticity is obtained using (10), derived in Section 3. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine, stratifying by notch, 100 times.
<table>
<thead>
<tr>
<th>Panel A: Friction Adjustment Factor $a$</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>Pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Dom. Region (Notched Banks)</td>
<td>0.11</td>
<td>0.15</td>
<td>0.15</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(2) Dom. Region (All Banks)</td>
<td>0.21</td>
<td>0.30</td>
<td>0.15</td>
<td>0.08</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(3) Entire Hole</td>
<td>0.67</td>
<td>0.60</td>
<td>0.57</td>
<td>0.40</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.09)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Elasticity of Intertemporal Substitution $\sigma$</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>Pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4) No Friction Adjustment</td>
<td>0.02</td>
<td>0.09</td>
<td>0.06</td>
<td>0.12</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.04)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(5) Dom. Region (Notched Banks)</td>
<td>0.02</td>
<td>0.12</td>
<td>0.08</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(6) Dom. Region (All Banks)</td>
<td>0.03</td>
<td>0.18</td>
<td>0.08</td>
<td>0.14</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(7) Entire Hole</td>
<td>0.17</td>
<td>0.54</td>
<td>0.34</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.09)</td>
<td>(10.87)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

Notes: The table shows how the estimated EIS is affected by assumptions on optimization frictions. The top panel shows the friction-adjustment factor $a$ estimated in three cases. Row (1) shows the estimate when restricting the sample to banks that have a non-zero interest rate notch. Row (2) shows our preferred estimate using the observed mass in the dominated region as given by equation (13). Row (3) calculates $a$ for the extreme assumption that all households in the hole above the notch are non-optimizers. The bottom panel shows the estimated EIS. Row (4) estimates the EIS without the friction adjustment; row (5) for notched banks only using the frictions adjustment from row (2); row (6) for the adjustment using the dominated region from row (3); and row (7) for the case that all households in the “hole” above the notch are non-optimizers as in row (4).
Table 4: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refinancing Cost</td>
<td>£1,000</td>
<td>Moneyfacts</td>
</tr>
<tr>
<td>House Price Process</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>0.875</td>
<td>Nationwide</td>
</tr>
<tr>
<td>$p_1$</td>
<td>0.006</td>
<td>mortgage data</td>
</tr>
<tr>
<td>$\sigma_p^2$</td>
<td>0.006</td>
<td>1974–2016</td>
</tr>
<tr>
<td>Quadratic lifecycle</td>
<td>linear</td>
<td>1,360</td>
</tr>
<tr>
<td>income profile coefficients</td>
<td>quadratic</td>
<td>14</td>
</tr>
<tr>
<td>Unemployment probability</td>
<td>5%</td>
<td>Historical average</td>
</tr>
<tr>
<td>Replacement Rate</td>
<td>60%</td>
<td>Benefit formulas</td>
</tr>
<tr>
<td>Future Bank of England policy rate</td>
<td>Calibrated to yield curve</td>
<td></td>
</tr>
<tr>
<td>Inflation expectations</td>
<td>2%</td>
<td>Bank of England target</td>
</tr>
<tr>
<td>Bequest motive</td>
<td>$\Gamma$ 0.1</td>
<td>Internally calibrated</td>
</tr>
<tr>
<td>Mortgage amortization rate</td>
<td>$\mu_t$ 1 / (70 - Age + 1)</td>
<td>Moneyfacts</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$ 2</td>
<td>Literature</td>
</tr>
<tr>
<td>Housing depreciation</td>
<td>$d$ 0.025/annum</td>
<td>Harding et al. (2007)</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\delta$ 0.96</td>
<td>Literature</td>
</tr>
</tbody>
</table>

Notes: This table shows calibrated parameters, their values, and source. A detailed description is found in Section 5.3.
Table 5: From Bunching to the EIS: Full Structural Model

<table>
<thead>
<tr>
<th>Statistic</th>
<th>60</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.96</td>
<td>2.94</td>
<td>6.86</td>
<td>6.42</td>
<td>7.45</td>
<td>4.11</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.26)</td>
<td>(0.39)</td>
<td>(0.74)</td>
<td>(0.99)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>$a$</td>
<td>0.58</td>
<td>0.21</td>
<td>0.30</td>
<td>0.15</td>
<td>0.08</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$b_{Adj}$</td>
<td>2.31</td>
<td>3.73</td>
<td>9.87</td>
<td>7.59</td>
<td>8.11</td>
<td>5.57</td>
</tr>
<tr>
<td></td>
<td>(0.49)</td>
<td>(0.35)</td>
<td>(0.60)</td>
<td>(0.89)</td>
<td>(1.16)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>$\Delta \lambda$</td>
<td>0.67</td>
<td>1.06</td>
<td>3.32</td>
<td>2.68</td>
<td>3.71</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.09)</td>
<td>(0.18)</td>
<td>(0.32)</td>
<td>(0.70)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>EIS $\sigma$</td>
<td>0.05</td>
<td>0.04</td>
<td>0.11</td>
<td>0.11</td>
<td>0.28</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.15)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

Notes: The table shows the results of our structural estimation as described in Section 5. $b$ is our normalized bunching estimate as described in Section 3 and the footnote to Figure 7. $a$ is the adjustment factor for optimization frictions (the number of individuals observed in the dominated region divided by the number of individuals in the same region in the counterfactual distribution), and $b_{Adj} = b / (1 - a)$ is our bunching estimate, adjusted for optimization frictions. $\Delta \lambda$ is the leverage response corresponding to $b_{Adj}$. $\sigma$ is the Elasticity of Intertemporal Substitution (EIS) that solves the full model described in Section 5. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine, stratifying by notch, 100 times.
<table>
<thead>
<tr>
<th>Covariate</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
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<tr>
<td>Age</td>
<td>0.06</td>
<td>0.13</td>
<td>0.15</td>
<td>0.19</td>
</tr>
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<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Household Income</td>
<td>0.02</td>
<td>0.06</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Loan to Income</td>
<td>0.03</td>
<td>0.06</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Income Growth</td>
<td>0.08</td>
<td>0.10</td>
<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>House Price Growth Rate</td>
<td>0.14</td>
<td>0.08</td>
<td>0.07</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Interest Rate Change (Passive)</td>
<td>0.10</td>
<td>0.06</td>
<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

Notes: The table shows the heterogeneity in our estimated EIS $\sigma$ (using the pooled average notch) by age, income, loan to income (LTI), income growth, house price growth, interest rate change since the previous mortgage (assuming passive borrower behavior). For each covariate, we partition the refiner panel into 4 quartiles and separately estimate $\sigma$ in each quartile. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine, stratifying by notch, 100 times.
Table 7: Robustness of EIS Estimates

<table>
<thead>
<tr>
<th></th>
<th>Discount Factor $\delta$</th>
<th>Present Bias Factor $\beta$</th>
<th>Risk Aversion $\gamma$</th>
<th>Future Interest Rates</th>
<th>House Price Growth</th>
<th>House Price Variance</th>
<th>Lifecycle Income Profile</th>
<th>Unemployment Probability</th>
<th>Replacement Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.7 0.9 0.96 0.99</td>
<td>0.3 0.5 0.7 1</td>
<td>0 1 2 CRRA</td>
<td>+0pp +1pp +2pp +3pp</td>
<td>-0.6% 0 0.6% 6%</td>
<td>0 0.004 0.006 0.008</td>
<td>£44K  £46K  £56K  £80K</td>
<td>3% 5% 7% 10%</td>
<td>60% 80% 100%</td>
</tr>
<tr>
<td>(2)</td>
<td>$\delta$</td>
<td>$\beta$</td>
<td>$\gamma$</td>
<td>Rates</td>
<td>Growth</td>
<td>Variance</td>
<td>Profile</td>
<td>Probability</td>
<td>Rate</td>
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<tr>
<td>(3)</td>
<td>0.13 0.12 0.08 0.12</td>
<td>0.17 0.14 0.11 0.08</td>
<td>0.11 0.12 0.08 0.15</td>
<td>0.08 0.11 0.12 0.12</td>
<td>0.10 0.10 0.08 0.10</td>
<td>0.08 0.10 0.08 0.16</td>
<td>0.12 0.08 0.09 0.08</td>
<td>0.09 0.08 0.11 0.12</td>
<td>0.08 0.13 0.12</td>
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<tr>
<td>(4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0% 0.7% 2.7% 6.5%</td>
<td>3% 5% 7% 10%</td>
<td>60% 80% 100%</td>
</tr>
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<td>(5)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table shows the robustness of our estimates of the elasticity of intertemporal substitution $\sigma$ to a number of the assumptions of our structural model. Panel (1) varies the discount factor $\delta$. Panel (2) relaxes our assumption that households discount the future with traditional geometric discounting, and allows for quasi-hyperbolic $\beta - \delta$ discounting with present bias parameters $\beta$ from 0.3 to 1 (geometric discounting). Panel (3) varies the coefficient of relative risk aversion $\gamma$. Panel (4) varies (deterministic) future interest rates by shifting the entire yield curve up by 1pp to 3pp. Panel (5) varies real house price growth ranging from house price declines to house price increases an order of magnitude larger. Panel (6) varies the variance of house price growth in household expectations. Panel (7) varies the deterministic lifecycle component of income expectations. In all cases, the lifecycle component is quadratic. The "Peak" parameter gives peak income and the "Slope" parameter gives expected income growth at the time of refinancing. Panels (8) and (9) vary assumptions on income uncertainty, with panel (8) varying the probability of unemployment and panel (9) the unemployment replacement rate.
Web Appendix (Not For Publication)

A Supplementary Figures & Tables

**Figure A.1: Refinancing Happens When the Reset Rate Kicks In**

Notes: The figure shows the distribution of the time to refinance, excluding individuals where the date on which the reset rate kicks in is unobserved. The figure shows individuals who refinance more than 6 months after their reset rate kicks in in black, individuals who refinance more than 2 months before their reset rate kicks in in white, and the remainder who refinance around their reset date in gray.
Figure A.2: Estimating Interest Rate Jumps With Borrower Demographics

Notes: The figure shows the conditional interest rate as a function of the Loan-To-Value (LTV) ratio based on a regression like (1), but adding controls for borrower demographics. Specifically, we add controls for age, income, single/couple status, and the reason for refinancing. In each LTV bin, we plot the estimated coefficient on the LTV bin dummy plus a constant given by the mean predicted value $E[\hat{r}_i]$ from all the other covariates. The figure shows that the mortgage interest rate evolves as a step function with sharp notches at LTV ratios of 60%, 70%, 75%, 80%, and 85%. These notches are virtually unchanged compared to the specification without borrower demographics.
Figure A.3: Equity Extraction by Passive LTV for Non-Bunchers

Notes: The figure shows the moving average of equity extracted on the y-axis, calculated among households that do not bunch in the actual LTV distribution. The x-axis is the passive LTV, i.e. the LTV that results from applying the amortization to the previous mortgage and using the new lender-assessed property valuation. This moving average is used to adjust the passive LTV distribution to obtain the counterfactual LTV distribution.
Figure A.4: Number of Past and Future Bunching Events by Current LTV

Notes: The figure shows the average number of past and future bunching events as a function of current LTV choice.
Figure A.5: No-Notch Sample and Round-Number Bunching

A: No-Notch Sample vs Full Sample

B: Zooming in on the No-Notch Sample

Notes: This figure shows frequency of refiners in the neighborhood of notches, in bank-month observations where the bank didn’t feature a interest rate jump at the notch. Panel A show this distribution alongside the frequency of refiners in the neighborhood of notches, in bank-month observations where a notch was present. It demonstrates that “no-notch banks” are a relatively small portion of our sample. Panel B zooms in on the distribution of mortgages at “no-notch banks”, together with the counterfactual distribution. It shows a small amount of round number bunching. We correct our estimates of bunching in response to interest rates with the magnitude of round number bunching.
Figure A.6: Interest Rate Schedules in Banks With and Without Notches

Notes: This figure shows the average interest rate in the neighborhood of the pooled notch for bank-months that featured a notch and those that did not. The interest rate is estimated using equation (1) for these subsamples. Relative to "no-notch banks", banks with notches offer a discount at LTVs below the notch. “notched"
Notes: The figure shows a representative simulation of the structural model for an EIS of $\sigma = 0.1$. The top panels give the exogenous paths of house prices (left panel) and income (right panel). The middle panels show the household’s choices of housing quality (left) and LTV (right). The bottom panels show the household’s choices of consumption (left) and liquid asset holdings (right). As the household accumulates wealth over the lifecycle, it increases housing quality, timing net housing purchases to housing price slumps. The LTV path shows bunching at both housing upgrades (at the 80% and 75% notches) and at several refinancing events. The household holds large liquid holdings at each refinance and draws down on these assets between mortgages. The household attempts to smooth consumption, but is faced by a negative income shock following a housing upgrade in the late 40s and hits the liquidity constraint, which leads to a significant consumption decline.
A.1 Proof of Propositions

A.1.1 Proposition 1

The Euler equation (4) and the budget constraints (2) and (3) imply:

\[ y_1 - R\lambda P_0 H + (1 - d) P_1 H = (\delta R)^\sigma [W_0 + y_0 - (1 - \lambda) P_0 H] . \]  \hspace{1cm} (A.1)

Applied to the marginal buncher at the counterfactual, this gives (5). Applied at the optimal interior LTV it gives

\[ \lambda^I P_0 H = \frac{y_1 + (1 - d) P_1 H - (\delta (R + \Delta R))^\sigma (W_0 + y_0 - P_0 H)}{(\delta (R + \Delta R))^\sigma + (R + \Delta R)} . \]

Then consumption in period zero at the optimal interior LTV is given by

\[ c_0^I = W_0 + y_0 - (1 - \lambda^I) P_0 H = \frac{1}{(\delta R)^\sigma} \left( (\delta R)^\sigma + R + \Delta R \right) \left( \frac{y_1}{\Pi_0} + (1 - d) \Pi_1 \right) - (R + \Delta R) \left( (\delta R)^\sigma + R \right) (\lambda^* + \Delta \lambda) (\delta \lambda) P_0 H . \]  \hspace{1cm} (A.2)

Using the Euler equation, the value of bunching at the interior is given by

\[ V^I = \frac{\sigma}{\sigma - 1} \left( 1 + \delta^\sigma (R + \Delta R)^{\sigma - 1} \right) \left( c_0^I \right)^{\frac{\sigma - 1}{\sigma}} . \]

Plugging (A.2) into this last equation gives the value of the best interior LTV in (6).

Using the budget constraints (2) and (3), with LTV at the notch, \( \lambda^* \), gives consumption of

\[ c_0^N = W_0 + y_0 - P_0 H + \lambda^* P_0 H = \frac{y_1}{\Pi_0} + (1 - d) \Pi_1 - R\lambda^* - ((\delta R)^\sigma + R) \Delta \lambda \frac{P_0 H}{(\delta R)^\sigma} . \]
and
\[ c_N^1 = y_1 - R\lambda^* P_0 H + (1 - d) P_1 H = \left( \frac{y_1}{P_0 H} + (1 - d) \Pi_1 - R\lambda^* \right) P_0 H. \]

Together, these give lifetime utility as in (7). The marginal buncher is defined as one who is indifferent between the optimal interior \( c_{(0)} = c_{(1)} \) LTV and bunching at the notch, so that \( V^N = V^I \), giving the statement in Proposition 1.

A.1.2 Proposition 2

At an optimal interior LTV choice, (2) to (4) give
\[ \lambda = \frac{y_1 + (1 - d) P_1 H + (\delta R)\sigma (P_0 H - W_0 - y_0)}{((\delta R)^\sigma + R) P_0 H}. \]

Differentiating this equation with respect to the interest rate \( R \) gives (10).

A.1.3 Proposition 3

Plugging \( R = \delta = 1 \) into (10) gives (12).

A.1.4 Proposition 4

As \( \sigma \to 0 \), the Euler equation gives \( c_1 = c_0 \). Lifetime utility converges to Leontief preferences and utility is equal to the smaller of \( c_0 \) and \( c_1 \). The Euler equation holds at the best interior LTV so that lifetime utility is given by period zero consumption \( c_0^I \). Bunching at the notch requires forgoing current consumption for future consumption, so that \( c_0^N < c_1^N \), and lifetime utility at the notch is given by \( c_0^N \). Thus households are better off bunching at the notch even with a zero EIS for counterfactual LTVs that give \( c_0^I < c_0^N \).

Applying the Euler equation \( c_1 = c_0 \) and the budget constraints (2) and (3) at the counterfactual with \( \sigma = 0 \) imply that initial wealth satisfies:
\[ W_0 + y_0 - P_0 H = y_1 + (1 - d) P_1 H - (R + 1) (\lambda^* + \Delta \lambda) P_0 H. \]
At this level of initial wealth $c^I_1 = c^I_0$ and the budget constraints imply that the best interior LTV is

$$\lambda^I = \frac{R + 1}{R + \Delta R + 1} (\lambda^* + \Delta \lambda).$$

Applying this to (2) gives period zero consumption of

$$c^I_0 = y_1 + (1 - d) P_1 H - \frac{(R + 1) (R + \Delta R)}{R + \Delta R + 1} (\lambda^* + \Delta \lambda) P_0 H.$$

Applying (2) at the notch, where $\lambda = \lambda^*$ and the interest rate is $R$ gives

$$c^N_0 = y_1 + (1 - d) P_1 H + (\lambda^* - (R + 1) (\lambda^* + \Delta \lambda)) P_0 H.$$

As noted above, a region of the counterfactual distribution is strictly dominated by bunching if $c^N_0 > c^I_0$ even as $\sigma \to 0$. Applying the last two equation to this inequality gives

$$\frac{\lambda^* + \Delta \lambda}{\lambda^*} \leq \frac{R + \Delta R + 1}{R + 1},$$

or

$$\lambda^* + \Delta \lambda < \left(1 + \frac{\Delta R}{R + 1}\right) \lambda^*,$$

giving the dominated range in (13).

A.2 Multi-period Model

The model in section 3, can easily be extended to have $T + 1$ periods. In this extended model, we assume households don’t face notches in the future, and constant house price growth net of depreciation $\Pi$. Households maximize their lifetime utility from non-housing consumption

$$\frac{\sigma}{\sigma - 1} \sum_{t=0}^{T} \delta^t (c^I_t)^{\frac{\sigma - 1}{\sigma}}$$

and face a sequence of budget constraints given by

$$c_t = \begin{cases} 
 y_0 + W_0 - (1 - \lambda_1) P_0 H_0 & \text{if } t = 0 \\
 y_t - R_t \lambda_t P_{t-1} H_{t-1} + \lambda_{t+1} P_t H_t & \text{if } 1 \leq t < T \\
 y_T - R_T \lambda_T P_{T-1} H_{T-1} + P_T H_T & \text{if } t = T
\end{cases} \quad (A.3)$$
In the absence of notches, household maximization yields the Euler equation

\[ c_t = (\delta R_t)^\sigma c_{t-1} \quad 1 \leq t \leq T - 1. \]  

(A.4)

Combining this with the budget constraints from period 1 onward, period 1 consumption satisfies

\[ c_1 = \frac{Y + (R_H - R_1 \lambda_1) P_0 H_0}{\bar{R}} \]  

(A.5)

where \( \bar{R} \equiv \sum_{t=1}^{T} (\delta^\sigma)^{t-1} \prod_{s=1}^{t-1} (R_{s+1})^{\sigma-1} \) is a sufficient statistic for the future path of interest rates, \( R_H \equiv (\prod_{s=2}^{T} R_s^{-1} \Pi^T) \) gives the value of house price appreciation to period \( T \), and \( Y \equiv \sum_{t=1}^{T} y_t \prod_{s=1}^{t-1} R_s^{-1} + \sum_{t=1}^{T} \delta R_t \sigma \) is the net present value of the household’s income from period 1 inwards. Note that if interest rates are constant at \( R \) these become \( \bar{R} = \left[ 1 - (\delta^\sigma R^{\sigma-1})^T \right] / \left[ 1 - (\delta^\sigma R^{\sigma-1}) \right], R_H = R^{T-1} \Pi^T, \) and \( Y = (1 - R^T) / (1 - R^{-1}) \).

To derive the indifference condition of the marginal buncher in the \( T + 1 \) period model, we start by analyzing the marginal bunching household’s counterfactual LTV choice at a constant interest rate \( R_1, \lambda^*_1 + \Delta \lambda_1 \). This choice satisfies the Euler equation (A.4) in period 1 and using (A.5) allows us to express the marginal bunching household’s wealth as a function of the other parameters of the model through

\[ W_0 = P_0 H_0 - y_0 + \frac{Y + \left[ \frac{R_H}{R} - \left( \frac{R_1}{R} + (\delta R_1) \sigma \right) (\lambda^*_1 + \Delta \lambda_1) \right] P_0 H_0}{(\delta R_1)^\sigma} \]  

(A.6)

The marginal buncher’s optimal interior choice of LTV \( \lambda^I \) at the higher interest rate \( R_1 + \Delta R \) also satisfies the Euler equation in period 1. Inserting the period-0 budget constraint (A.3), the period-1 budget constraint (A.5) and the expression for wealth (A.6) yields

\[ \lambda^I_1 P_0 H = \frac{Y + \frac{R_H}{R} P_0 H_0 - [\delta (R_1 + \Delta R)]^\sigma (y_0 + W_0 - P_0 H)}{\frac{R_1 + \Delta R}{R} + [\delta (R_1 + \Delta R)]^\sigma} \]  

(A.7)

Inserting equations (A.6) and (A.7) into the period-0 budget constraint, this choice of LTV yields consumption of
\[ c_0^I = y_0 + W_0 - P_0H + \lambda_1^I P_0H \]

\[ = \left( \frac{Y}{R} + \frac{R_H P_0H_0}{R} \right) \left[ (\delta R_1)^\sigma + \left( \frac{R_1 + \Delta R}{R} \right) - (\lambda_* + \Delta \lambda_1) \right] \frac{R_1 + (\delta R_1)^\sigma}{(\delta R_1)^\sigma} \]

\[ = \left( \frac{Y}{R} + \frac{R_H P_0H_0}{R} \right) \left[ (\delta R_1)^\sigma + [\delta (R_1 + \Delta R)]^\sigma \right] \]

(A.8)

and so the lifetime non-housing consumption utility of the marginal buncher at the interior choice \( \lambda^I \) is given by

\[ V^I \]

\[ = \frac{\sigma}{\sigma - 1} \left[ \left( c_0^I \right)^{\frac{\sigma - 1}{\sigma}} + \delta \tilde{R} (c_1^I)^{\frac{\sigma - 1}{\sigma}} \right] = \frac{\sigma}{\sigma - 1} \left[ (c_0^I)^{\frac{\sigma - 1}{\sigma}} + \delta \tilde{R} \left( [\delta (R_1 + \Delta R)]^\sigma \right) \right] \]

\[ = \frac{\sigma}{\sigma - 1} \left( \frac{R_1 + \Delta R}{R} \right) + [\delta (R_1 + \Delta R)]^\sigma \right] \frac{1}{\sigma} \] \( (\delta R_1)^{1-\sigma} \]

\[ \times \left( \left[ \frac{Y}{R} + \frac{R_H P_0H_0}{R} \right] \left[ (\delta R_1)^\sigma + \left( \frac{R_1 + \Delta R}{R} \right) \right] - (\lambda_* + \Delta \lambda) \right) P_0H_0 R_1 + \Delta R \left[ \frac{R_1}{R} + (\delta R_1)^\sigma \right] \] \( \frac{\sigma - 1}{\sigma} \]

(A.9)

If instead the marginal buncher chooses to be at the notch, the household’s period-0 consumption is gets

\[ c_0^* = y_0 + W_0 - P_0H + \lambda_1^* P_0H \]

\[ = \frac{Y}{R} + \frac{R_H - \left( \frac{R_1}{R} + (\delta R_1)^\sigma \right) \left( \lambda_*^* + \Delta \lambda_1 \right) P_0H_0 + \lambda^* P_0H_0 (\delta R_1)^\sigma}{(\delta R_1)^\sigma} \]

(A.10)

where the second equality follows by substituting wealth using equation \((A.6)\). Equation \((A.5)\) implies that their period-1 consumption is

\[ c_1^* = \frac{Y + (R_H - R_1 \lambda_*^*) P_0H_0}{R} \]

(A.11)

giving lifetime consumption utility of

\[ V^N = \frac{\sigma}{\sigma - 1} \left[ \left( \frac{Y}{R} + \frac{R_H - \left( \frac{R_1}{R} + (\delta R_1)^\sigma \right) \left( \lambda_*^* + \Delta \lambda_1 \right) P_0H_0 + \lambda^* P_0H_0 (\delta R_1)^\sigma}{(\delta R_1)^\sigma} \right)^{\frac{\sigma - 1}{\sigma}} \]

\[ + \delta \tilde{R} \left( \frac{Y + (R_H - R_1 \lambda_*^*) P_0H_0}{R} \right)^{\frac{\sigma - 1}{\sigma}} \]

(A.12)
The EIS in the extended model is therefore the solution to $V^N = V^I$.

We can also derive the reduced-form elasticity $\varepsilon$ in the multi-period model by differentiating the period-1 Euler equation with respect to the period-1 interest rate (holding all future interest rates constant), yielding

$$
\varepsilon \equiv -\frac{d \log \lambda_1}{d \log R_1} = \frac{(R_1 / \bar{R}) + \sigma (\delta R_1)^\sigma}{(R_1 / \bar{R}) + (\delta R_1)^\sigma} - \frac{\sigma (\delta R_1)^\sigma}{1 + (\delta R_1)^\sigma} \frac{P_0 H_0 - y_0 - W_0}{(Y + R_H P_0 H_0) / R} \tag{A.13}
$$

As $\sigma \to 0$, $\varepsilon \to \frac{R_1 / \bar{R}}{(R_1 / \bar{R}) + 1} \simeq \frac{1}{1 + T}$ bounding $\varepsilon$ from below in the generalized model.