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Optimum taxation and the allocation of time

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Abstract

A theory of optimal taxation is presented, building upon Becker's (1965) theory of the allocation of time. Optimal commodity taxation is governed by factor shares in household activities. Any market good which requires little household time, or even saves time, should carry a relatively low tax rate. This policy rule does not require the estimation of price elasticities and is therefore more applicable than traditional Ramsey rule taxation.

JEL classification: H21; D13; J22
Keywords: Commodity taxation; Household production; Time allocation

1. Introduction

In the theory of optimum taxation, recently surveyed by Auerbach and Hines (2002), commodity taxes are governed by the so-called Ramsey rule which emphasizes the importance of compensated price responses. Unfortunately, the applicability of Ramsey rule taxation is hampered by the fact that little is known about the magnitudes of the relevant elasticities. Indeed, many have argued that the empirical evidence needed for the elimination of this ignorance is simply not obtainable. This paper demonstrates, however, that the problem of applicability may be alleviated once we account for the presence of household production.

Optimal tax theory is based on the labor–leisure model, which assumes that households derive utility from the consumption of market-produced goods and household time,

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separately. But as pointed out by Becker (1965), Lancaster (1966), and Muth (1966) this is not a realistic description of behavior. The transformation of market goods into utility takes time and, conversely, time per se does not yield utility without the use of complementary market inputs. In other words, utility-yielding commodities take the form of activities using both market goods and time. For example, an activity such as going to the movies requires the purchase of tickets, transportation, etc., as well as the moviegoer's time. We may think of this as a theory of consumption technology or, alternatively, as a theory of household production although it does not deal with production activities in the common sense of the term.

Ideally, the social planner wishes to tax the fixed time endowment of households thereby retaining the first best allocation. In the labor–leisure model, this corresponds to a uniform tax on all commodities including pure leisure. But once the taxation of leisure is ruled out, as in the Ramsey tax problem, it is no longer possible to achieve first best in this simple way. Instead, the optimal policy rule tries to compensate for the missing instrument by taxing those goods which are complementary to leisure. However, this creates new distortions implying that the allocation is only second best.

In the theory presented here, building upon the Becker (1965) framework, the solution is surprisingly simple. In this approach the absent tax on household time may be compensated for by taxing those market goods used to transform time into utility. In other words, we may not be able to tax leisure directly but because leisure take the form of activities we may do so indirectly. For example, an excise on movie tickets indirectly taxes the activity 'going to the movies', an excise on golf clubs taxes the activity 'playing golf', etc.

Each market good may be characterized by a factor share, i.e., its cost share in the household activity using the good. The optimal tax system is given by an *inverse factor share rule*: the tax rate on any given market good is inversely proportional to its factor share. Equivalently, because the cost share for goods is equal to one minus the cost share for time, tax rates are positively related to time intensities. The optimal policy ensures that household activities are taxed at a uniform rate, thereby imitating a tax on the fixed time endowment which is first best.

The paper also explores the implications of introducing pure leisure into the Becker model. Since the activity using no market goods becomes effectively untaxable, uniform taxation of all activities is not possible and the allocation can no longer be first best. Yet, the inverse factor share rule still plays an important role for tax policy. In fact, if pure leisure is weakly separable in utility, the inverse factor share rule fully characterizes the optimal solution. More generally, optimal taxation in the Becker model turn out to be a simple transform of the Ramsey-type tax formulae, where the traditional rules are simply multiplied by the inverse factor share rule.

An important difference between the Ramsey-type rules and the inverse factor share rule proposed here lies in the information needed for their implementation. Ramsey rule taxation requires the estimation of compensated own and cross price elasticities. Moreover, we need to know the magnitude of these elasticities in the optimal point, which is presumably different from the current position or anything else previously observed. As argued by Deaton (1981, 1987), this kind of global knowledge of preferences and demand is not likely to come together. By contrast, the inverse factor share rule does not require the estimation of elasticities.

Any market good which requires little household time, or even saves time, should carry a relatively low rate of tax, and vice versa. This result provides a strong intuition for the structure of taxation, in contrast to the traditional emphasis on compensated price responses. The implications for the taxation of e.g. services, transportation, alcohol, and tobacco are discussed in the paper. Interestingly, Atkinson and Stern (1980, 1981) estimate factor shares in the Becker model, although they do not draw the implications for tax policy. Their results indicate that the optimal tax system is strongly non-uniform. The highest taxes (on alcohol and tobacco) exceed the lowest taxes (on services) by almost factor 10.

The household production approach has been put to wide use in the analysis of e.g. fertility, health, labor supply, and transportation. Quite surprisingly the approach has had relatively little impact on the theory of taxation, although a few exceptions deserve mentioning. Sandmo (1990) and Kleven et al. (2000) analyze the optimal tax treatment of market-produced services vis-à-vis other goods, taking into account that services supplied from the market may alternatively be produced within the home where they cannot be taxed. They do not adopt a true Becker model, however, but the much more specialized structure of Gronau (1977). It turns out that a more orthodox Becker approach yields even stronger policy recommendations than does the Gronau specification.

The study of Gahvari and Yang (1993) is closer in spirit to the present paper. Yet, they retain the assumption of the standard Ramsey analyses that one household activity is pure leisure which is non-separable in utility. In this case, the conditions characterizing the optimum are generally quite complex, and their implementation requires the knowledge of both demand elasticities and household production technology. The contribution of the present paper, on the other hand, is to derive the simple inverse factor share rule for optimal taxation and to demonstrate that it may imitate a tax on the fixed time endowment.

The paper is organized in the following way. Section 2 sets up the model of household production, while Section 3 derives the optimal tax system in the model version without pure leisure. Section 4 then introduces pure leisure, and consider optimal taxation under three popular preference structures. Section 5 discusses the policy implications of the analysis and, finally, Section 6 concludes.

2. The framework

A representative household derives utility from the consumption of commodities, Z^0, $Z^1,\ldots,Z^n$, that is

$$U = U\left(Z^0, Z^1, \ldots, Z^n\right). \tag{1}$$

Following Becker (1965), these Z commodities take the form of activities using both goods and time. Formally,

$$Z^i = f^i\left(X^i, L^i\right), \quad i = 0, 1, \ldots, n, \tag{2}$$

where X^i is the input of a market-produced good and L^i is the input of household time in the production of commodity i.[1] Let the coefficients $a_X^i \equiv X^i/Z^i$ and $a_L^i \equiv L^i/Z^i$ denote the input of goods and time, respectively, per unit of commodity i. I assume that these coefficients are fixed.

The use of a fixed coefficients consumption technology is standard in applications of the Becker framework. This type of technology is sufficiently flexible to retain the labor–leisure analysis as a special case. The labor–leisure model arises in the case where one activity uses no goods, say $a_X^0 = 0$, while all the other activities use no time, that is $a_L^i = 0$ for $i = 1, \ldots, n$. The framework also encompasses popular models of household production, for example the Gronau (1973, 1977) specifications employed in the optimal tax literature by Sandmo (1990) and others.

Note, furthermore, that the assumption of fixed coefficients in Eq. (2) does not rule out the possibility of substitution in household production. Basic to the notion of any production function is the existence of different production processes, where each process uses a certain fixed ratio of inputs. A production function, in other words, is a description of substitution possibilities between production processes. In this interpretation, we may think of the Z commodities as production processes, while the U function captures the possibilities for substitution between these processes.[2]

Household decisions must be made in accordance with the budget constraint

$$\sum_{i=0}^{n} P^i X^i - N = 0, \tag{3}$$

where N denote labor supply to the market, while P^i is the consumer price of good i. The consumer wage is normalized to one. Taxes are introduced by defining $P^i \equiv p^i + t^i P^i$, where p^i refers to the producer price and where t^i is the tax rate measured in proportion of the tax-inclusive price P^i. We disregard taxes on market labor income. We may do so without loss of generality because of the equivalence between a labor income tax and a uniform value-added tax on market goods. The real restriction on the tax system lies in the assumption, implicit in Eq. (3), that the value of household time is untaxable. This corresponds of course to the assumption made in the standard model that leisure is not taxed.

Decisions are also subject to the following time constraint

$$\sum_{i=0}^{n} L^i + N = 1, \tag{4}$$

[1] More generally, we may think of X^i as a composite input incorporating many different market-produced goods. It is possible that some of these market-produced goods enter more than one household activity. The only substantive assumption needed for our purpose is the feasibility of selective taxation across different X^i's.

[2] For example, dish-washing may be carried out by the use of a brush or a machine. These two production processes involve fixed (but different) ratios between time and market goods. Washing up with a brush and a machine, respectively, are then modelled as two different Z commodities, while the substitution between them lies in the U function.

where the total time available is normalized to one. The budget and time constraints may be combined so as to give a single overall constraint:

$$\sum_{i=0}^{n} Q^i Z^i = 1, \quad Q^i \equiv P^i a_X^i + a_L^i, \tag{5}$$

where Q^i denotes the cost per unit of commodity i, including the opportunity cost of time. This condition states that the total cost of consumption must be equal to so-called full income, i.e., the market value of the time endowment.

We solve the dual consumer problem by minimizing the LHS of (5) subject to $U \geq \bar{U}$ and (1). This expenditure minimization problem is formally similar to the standard one but, essential for the analysis of optimum taxation, the definition of prices are different. The solution is characterized by

$$\tilde{Z}^i = Z^i\left(Q^0, \ldots, Q^n, \bar{U}\right), \quad \tilde{X}^i = a_X^i \tilde{Z}^i, \quad \tilde{L}^i = a_L^i \tilde{Z}^i, \tag{6}$$

where a tilde refers to compensated demand or supply. Inserting compensated demands on the LHS of (5), we obtain the expenditure function $e\left(Q^0, \ldots, Q^n, \bar{U}\right)$.

3. Optimal taxation without pure leisure

To derive the optimal tax system, we define the dead-weight burden of taxation D as the equivalent variation minus the tax revenue, i.e.

$$D = e\left(Q^0, \ldots, Q^n, \bar{U}\right) - e\left(q^0, \ldots, q^n, \bar{U}\right) - T\left(P^0, \ldots, P^n, \bar{U}\right), \tag{7}$$

where q^i denotes the unit cost in activity i in the absence of taxation, while $\bar{U}$ is the after tax utility level. The total tax revenue $T(\cdot)$ is given by

$$T\left(P^0, \ldots, P^n, \bar{U}\right) = \sum_{i=0}^{n} t^i P^i \tilde{X}^i = \sum_{i=0}^{n} \left(P^i - p^i\right) a_X^i \tilde{Z}^i. \tag{8}$$

On the usual assumption of constant producer prices, the optimal tax system may be found by minimizing D with respect to consumer prices $P^0, \ldots, P^n$, accounting for the relation between consumer prices and unit costs given in Eq. (5). The minimization must be carried out subject to the government budget constraint, $T\left(P^0, \ldots, P^n, \bar{U}\right) = T$, where T is an exogenous revenue requirement. Defining the cost share for goods, $\alpha_X^i \equiv P^i a_X^i / Q^i$, we may state the following proposition:

Proposition 1. *If goods shares are positive in all activities, $\alpha_X^i > 0$ $\forall i$, then the first best optimal tax system is characterized by $t^i = T/\alpha_X^i$ $\forall i$.*

Proof. See Appendix A. □

Optimal taxation is characterized by an inverse factor share rule. The higher the goods share in a given activity, the lower the tax rate on market goods used in that activity. As the goods share is equal to one minus the time share, Proposition 1 alternatively implies that optimal tax rates are positively related to time intensities.

The intuition for our result is easily grasped once it is recognized that a first best tax system taxes the fixed time endowment of households. It is immediately seen from the budget/time constraint (5) that this would correspond to a uniform tax on household activities or, equivalently, a uniform tax on all market goods and household time. However, the social planner cannot tax activities or household time per se, only market goods are taxable. Yet, since the transformation of time into utility always requires goods, he may imitate a uniform tax on activities by employing the inverse factor share rule stated in Proposition 1.[3]

In the labor–leisure model, on the other hand, sustaining the first best allocation would require a proportional tax on all commodities including pure leisure. Once the taxation of pure leisure is ruled out, as in the Ramsey tax problem, there is no way to circumvent the lack of an instrument. We may try to compensate for the missing tax on leisure by taxing its complements—as reflected by the Ramsey-type tax rules—but this introduces new distortions implying that the allocation is only second best.

4. Optimal taxation with pure leisure

Proposition 1 assumes that the taxation of market-produced goods is not restricted. But in practice certain goods may be untaxable for reasons such as administrative costs, imperfect information, or political inefficiencies. If some market goods are not taxed then neither are the household activities using these goods. In this case it is not possible to impose uniform taxation of all activities by employing the inverse factor share rule. A simple form of restriction is where one market good, say no. 0, cannot be taxed, such that the corresponding activity Z^0 is also untaxed. This situation may be captured in a simple way by modelling Z^0 as if it is pure leisure, i.e. $a_X^0 = \alpha_X^0 = 0$.

First, consider a specific preference structure which features quite prominently in the public finance literature, namely one where utility is weakly separable in pure leisure Z^0 and homothetic in non-leisure commodities $Z^1, \ldots, Z^n$. In the standard framework, where

[3] The simplicity of the inverse factor share rule clearly relies on the assumption of Leontieff technology. As mentioned in Section 2 this assumption may be justified by thinking of the Z-activities as production processes. However, in this interpretation one would naturally expect the consumer's optimum to be characterized by $Z^i = 0$ for some subset of processes. Although the above analysis assumes that all activities are carried out at positive levels, Proposition 1 would go through in the presence of corner solutions. The inverse factor share rule then applies to actual *and* potential household activities. The policy rule does not change relative unit costs between processes, and those processes which are unused in the presence of taxation would also be unused in the absence of taxation. Admittedly, the optimal tax system in the above story would involve a lot of different rates and be quite difficult to implement for administrative reasons. This stresses the importance of the following section which considers the case of untaxable commodities. Rather than introducing untaxable activities, one may alternatively follow the route of Kleven (2000) who analyses a more general household production technology with substitution.

non-leisure commodities are purchased directly in the market, these preferences imply that the second best optimum is characterized by uniform taxation of market goods (see e.g. Deaton and Muellbauer, 1980; Deaton, 1981). In the Becker model, on the other hand, this result translates into uniform taxation of activities rather than market goods. And this in turn gives rise to the inverse factor share rule. Defining required revenue in proportion to the tax base, $T' \equiv T/\left(1-\tilde{L}^0\right)$, we get the following result:

Proposition 2. *Consider a preference structure $U=U(Z^0,G(Z^1,\ldots,Z^n))$ where Z^0 is pure leisure and where G is a homothetic function. Then the second best optimal tax system is characterized by $t^i=T'/\alpha_X^i$ for $i=1,\ldots,n$.*

Proof. See Appendix B. □

Thus, relative tax rates are identical to Proposition 1 but, since one activity is untaxable, the level of tax rates have to be adjusted so as to collect an unchanged revenue T. The allocation is now second best because the commodity tax system distorts household behavior in favor of the untaxed activity.

Now, consider the case where pure leisure is not separable in utility. In this case standard Ramsey taxation is no longer flat, but depends on compensated own and cross price responses of all the different market goods. One would expect that the policy rule in the Becker model also becomes a function of these price responses and, indeed, this turn out to be the case. Yet, I shall demonstrate below that the inverse factor share rule still plays an important role for optimal taxation.

In general, optimal policy in the Becker model may be represented in a way which is reminiscent of the traditional Ramsey rule, except that the policy rule is expressed in terms of household activities rather than market goods (see Eq. B.2 in Appendix B). Unfortunately, the Ramsey rule in its most general form provides little guidance on tax policy. In providing such guidance, the literature has usually proceeded to investigate various special cases which give rise to intuitive elasticity formulae for optimal tax rates. I will reconsider two such cases, namely that of zero cross price effects between taxed goods and the three-commodity example of Corlett and Hague (1953).

Letting $\eta^{ji} \equiv (A\tilde{Z}^j/AQ^i)(Q^i/\tilde{Z}^j)$ denote the compensated cross elasticities between commodity j and i, and letting μ denote the Lagrange multiplier associated with the government budget constraint, we may state the following propositions:

Proposition 3. *Consider a preference structure $U=U(Z^0,Z^1,\ldots,Z^n)$ where Z^0 is pure leisure and where there are no cross price effects between $Z^1,\ldots,Z^n$, i.e., $\eta^{ji}=0$ for $i\neq j$ and $i,j=1,\ldots,n$. Then the second best optimal tax system is characterized by*

$$t^i = -\left[\frac{\mu}{1+\mu}\frac{1}{\eta^{ii}}\right]\frac{1}{\alpha_X^i} \quad i=1,\ldots,n.$$

Proof. See Appendix C. □

Proposition 4. *Consider a preference structure $U=U(Z^0,Z^1,Z^2)$ where Z^0 is pure leisure. Then the second best optimal tax system is characterized by*

$$\frac{t^1}{t^2}=\left[\frac{\eta^{11}+\eta^{22}+\eta^{10}}{\eta^{11}+\eta^{22}+\eta^{20}}\right]\frac{\alpha_X^2}{\alpha_X^1}.$$

Proof. See Appendix D. □

The bracketed term in Proposition 3 is an inverse elasticity rule, while the bracketed term in Proposition 4 is a Corlett–Hague rule (see e.g. Myles, 1995, chapter 4), except that the above elasticities reflect the responsiveness of activities rather than market goods. These propositions show that the inverse factor share rule survives in a ceteris paribus sense. Even if we are ignorant about elasticities, factor shares may indicate how selective commodity taxation could be used to improve efficiency. But to implement *optimal* tax rates in the presence of non-separable pure leisure would require information on both elasticities and factor shares.

5. The inverse factor share rule in practice

According to the theory presented here, any type of consumption which uses little time, or even saves time, should carry a relatively low rate of tax. Introspection immediately leads to a couple of obvious candidates. Services, for example, have low time intensities almost by definition: hiring somebody in the market to supply a service, rather that doing it yourself, saves household time. Accordingly, goods such as child care, cleaning, housekeeping, cooking, dish-washing, garden care, house- and car-repair should be taxed leniently or even subsidized. Indeed, such a policy has been undertaken in several EU countries in recent years. Now we have a rational argument for doing so.

Transportation constitutes another interesting example. Fast transportation should carry a lower rate of tax than slow transportation, since the former saves time and requires more expensive market goods. This recommendation clashes with the widespread policy of imposing high taxes on gasoline and cars. Of course, one should be cautious here because the paper does not account for the presence of negative externalities but, nevertheless, our result does indicate that the efficiency cost of the current policy may be higher than previously thought.

Furthermore, goods which promote health are likely to save or create time: they reduce time off work and may lengthen the working life. Thus, healthy goods should be subsidized whereas unhealthy goods, e.g. alcohol and tobacco, should carry relatively high taxes. Notice that this argument is fundamentally different from traditional arguments relying on the presence of externalities or the idea of merit wants.

These examples demonstrate the strength of the Becker approach. While the traditional focus on own and cross price elasticities provides no intuition on the optimal taxation of, say, services or transportation, these are clear-cut cases in the analysis presented here. Yet, tax reform ought to be guided by hard evidence on the relevant parameters rather than introspective arguments. An important advantage of the inverse factor share rule is that its implementation requires no econometric estimation, at least not in principle. A combined survey of consumption expenditures and time allocation would enable a direct measurement of factor shares and provide the basis for tax policy.

In lack of data on the allocation of time, one may alternatively follow the route of Atkinson and Stern (1980, 1981) who estimate factor shares in the Becker model. Although this procedure involves the same problems as the estimation of price elasticities, it is interesting that the Atkinson–Stern results seem to confirm the above conclusions. They find that the consumption of services is the least time intensive activity, while alcohol and tobacco are the most time intensive. Inserting the estimated factor shares into the inverse factor share rule implies a strongly non-uniform structure. If the government revenue requirement is, for example, 20 percent of income, optimal commodity taxes vary from 14 percent to 118 percent.

6. Conclusion

It is commonly argued that the economics literature has taught us relatively little about commodity tax policy, e.g. Harberger (1988) and Boadway (1997). The conditions characterizing the optimum turn out, in general, to provide little intuition on tax policy. Attempts to interpret the conditions as proportional output reduction rules or inverse elasticity rules rely on unreasonable simplifications. Moreover, the information needed for optimal policy, essentially global knowledge of preferences and demand, is simply not obtainable.

I believe the theory of optimal taxation presented here is less vulnerable to the above critiques. The inverse factor share rule is simple and intuitive. It is optimal to tax market goods in inverse proportion to their cost shares in household activities, because such a policy avoids distortionary effects on the full relative prices of activities. The implementation of this rule does not require the estimation of price responses. A combined survey of consumption expenditures and time allocation would enable a direct measurement of the relevant factor shares and provide the basis for tax policy.

It seems that a majority of economists and policy makers favor uniform commodity taxation. Their view is partly justified by the optimal taxation literature. After all uniform taxation is the optimal solution under certain conditions, namely weak separability of leisure and homotheticity of goods. These conditions may be special but given our current knowledge they cannot be falsified, see e.g. Deaton (1981, 1987). Once we account for the presence of household production, however, it becomes difficult to maintain this position. Uniformity of taxation requires that factor shares are identical across activities but, clearly, this is not the case in reality. Indeed, there seems to be great variation across activities implying a strongly non-uniform optimal tax structure. Consequently, the welfare cost of uniform taxation could be quite high.

A most relevant extension would be the introduction of heterogeneity across households. Interestingly, the inverse factor share rule immediately generalizes to a simple form of heterogeneity, popular in the literature on optimum income taxation. In particular, consider the case where agents differ only in their market productivity. Then we may apply the Atkinson and Stiglitz (1976) proposition stating that the optimum involves uniform taxation of consumption goods, provided that leisure is weakly separable in utility. In the Becker context, however, this result translates into a uniform tax on household activities which gives us the inverse factor share rule.

This Atkinson–Stiglitz–Becker proposition has interesting implications for the long-standing controversy on whether capital should be taxed or not. The result suggests that the optimal taxation of consumption over the life cycle is related to the time intensity of consumption in different periods. Since old people spend relatively little time working in the market, their consumption basket will be more intensive in the use of household time than the consumption of people in their working age. Consequently, the consumption of old people should carry a relatively high rate of tax which is equivalent to a positive tax on savings.

Of course the Atkinson–Stiglitz setup may be too simple. If agents differ also in their preferences and household production technologies the analysis becomes more involved. In particular, one would like to generalize the inverse factor share rule to the case where factor shares vary across consumers. I leave this as a topic for future research.

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Appendix A. Proof of Proposition 1

First, by inserting $t^i = T/\alpha_X^i$, $\alpha_X^i \equiv P^i a_X^i / Q^i$ and Eqs. (5)–(6) in the expression for total tax revenue (8), it is seen that the government's revenue requirement is met, i.e., $T\left(P^0, \ldots, P^n, \bar{U}\right) = T$. Next, by inserting $t^i = T/\alpha_X^i$ in the definition of Q^i, we get $Q^i = q^i/(1-T)$. Therefore, the deadweight burden of taxation becomes

$$\begin{aligned} D &= e\left(q^0/(1-T), \ldots, q^n/(1-T), \bar{U}\right) - e\left(q^0, \ldots, q^n, \bar{U}\right) - T\left(P^0, \ldots, P^n, \bar{U}\right) \\ &= \frac{T}{1-T} e\left(q^0, \ldots, q^n, \bar{U}\right) - T\left(P^0, \ldots, P^n, \bar{U}\right), \end{aligned}$$

where we have exploited that the expenditure function is homogeneous of degree one. Moreover, using $Q^i = q^i/(1-T)$ and Eq. (5), we have $e\left(q^0, \ldots, q^n, \bar{U}\right) \equiv \sum_{i=0}^n q^i \tilde{Z}^i = 1 - T$. Thus, $D = T - T\left(P^0, \ldots, P^n, \bar{U}\right) = 0$, implying that the allocation is first best. □

Appendix B. Proof of Proposition 2

Since Z^0 uses no market goods (i.e. $a_X^0 = 0$), we wish to minimize D with respect to prices $P^1,\ldots,P^n$ subject to $T\left(P^1,\ldots,P^n,\bar{U}\right) = T$. Letting μ denote the Lagrange multiplier associated with the government budget constraint, the first-order conditions to this problem are given by

$$\frac{AD}{AP^j} = \mu\frac{AT}{AP^j} \qquad j = 1,\ldots,n, \tag{B.1}$$

along with the government budget constraint. From (7), we get

$$\frac{AD}{AP^j} = \tilde{Z}^j a_X^j - \frac{AT}{AP^j},$$

where we have used $Ae/AQ^j = \tilde{Z}^j$ and $AQ^j/AP^j = a_X^j$. Moreover, differentiation of (8) gives

$$\frac{AT}{AP^j} = \sum_{i=1}^{n} t^i P^i a_X^i a_X^j \frac{A\tilde{Z}^i}{AQ^j} + a_X^j \tilde{Z}^j.$$

Now, by inserting these derivatives in Eq. (B.1), and using the symmetry of the Slutsky matrix, i.e. $A\tilde{Z}^i/AQ^j = A\tilde{Z}^j/AQ^i$, we may write the first-order conditions in the following way

$$\sum_{i=1}^{n} t^i \alpha_X^i \eta^{ji} = -\frac{\mu}{1+\mu} \quad j = 1,\ldots,n, \tag{B.2}$$

where $\eta^{ji} \equiv (A\tilde{Z}^j/AQ^i)(Q^i/\tilde{Z}^j)$ denotes the compensated cross elasticities between commodity j and i, and where $\alpha_X^i \equiv P^i a_X^i/Q^i$ is the cost share for goods.

Consider the preference structure $U = U(Z^0, G(Z^1,\ldots,Z^n))$ where G is a homothetic function. This utility function implies that $\eta^{j0} = \eta$ for $j = 1,\ldots,n$, see e.g. Deaton and Muellbauer (1980, p. 128). Furthermore, because $\tilde{Z}^j$ is homogeneous of degree zero, we have that $\sum_{i=1}^{n} \eta^{ji} = -\eta^{j0} = -\eta$ for $j = 1,\ldots,n$. Exploiting these properties, it is seen that a solution to the first-order conditions in (B.2) is characterized by $t^i = T'/\alpha_X^i$ for $i = 1,\ldots,n$, and $\mu = \eta T'/(1-\eta T')$.

Finally, we need to determine the level of T' which is consistent with the government's revenue requirement. By inserting $t^i = T'/\alpha_X^i$, $\alpha_X^i \equiv P^i a_X^i/Q^i$, and $a_X^0 = 0$ in (8), it is seen that the government's revenue requirement implies $T' = T/\left(\sum_{i=1}^{n} Q^i \tilde{Z}^i\right)$. Using that $\sum_{i=1}^{n} Q^i \tilde{Z}^i = 1 - Q^0\tilde{Z}^0 = 1 - \tilde{L}^0$, we get $T' = T/\left(1 - \tilde{L}^0\right)$ as in Proposition 2. □

Appendix C. Proof of Proposition 3

By inserting $\eta^{ji} = 0$ for $i \neq j$ and $i,j = 1,\ldots,n$, the first-order conditions in (B.2) reduces to $t^j \alpha_X^j \eta^{jj} = -\mu/(1+\mu)$. Changing the index from j to i, this equation corresponds to the optimal tax rule in Proposition 3. □

Appendix D. Proof of Proposition 4

Consider now the case of three commodities Z^0, Z^1, Z^2, where Z^0 uses no market goods. The first-order conditions in (B.2) may then be written as

$$t^1\alpha_X^1\eta^{11} + t^2\alpha_X^2\eta^{12} = -\frac{\mu}{1+\mu},$$

$$t^1\alpha_X^1\eta^{21} + t^2\alpha_X^2\eta^{22} = -\frac{\mu}{1+\mu}.$$

Using Cramer's rule, we get

$$t^1 = -\frac{1}{\Delta}\frac{\mu}{1+\mu}\left[\eta^{22} - \eta^{12}\right]\alpha_X^2,$$

$$t^2 = -\frac{1}{\Delta}\frac{\mu}{1+\mu}\left[\eta^{11} - \eta^{21}\right]\alpha_X^1,$$

where $\Delta = \alpha_X^1\alpha_X^2[\eta^{11}\eta^{22} - \eta^{12}\eta^{21}]$. Relative tax rates are then given by

$$\frac{t^1}{t^2} = \left[\frac{\eta^{22} - \eta^{12}}{\eta^{11} - \eta^{21}}\right]\frac{\alpha_X^2}{\alpha_X^1}. \tag{D.1}$$

Now, because $\tilde{Z}^j$ is homogeneous of degree zero, we have that $\eta^{10} + \eta^{11} + \eta^{12} = 0$ and $\eta^{20} + \eta^{21} + \eta^{22} = 0$. Inserting these relations in Eq. (D.1), the optimal tax rule may be written on the form in Proposition 4. □

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