SUFFICIENT STATISTICS REVISITED

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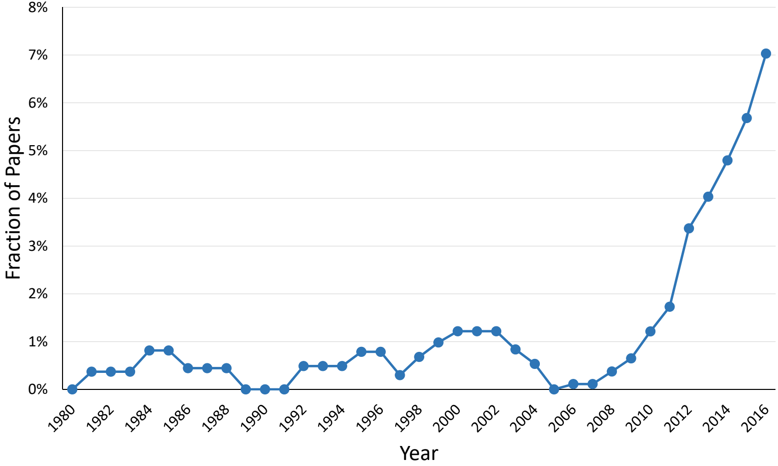
Sufficient Statistics Approach

- The idea is that the welfare effect of policy changes can be written in terms of estimable reduced-form elasticities
- This allows for policy evaluation without estimating the structural primitives of fully specified models
- A bridge between reduced-form approaches (credible identification) and structural approaches (welfare predictions)
- Chetty (2009) coined the phrase, but the intellectuel origins are very old
  - DWL: Harberger (1964), Feldstein (1999), Kleven-Kreiner (2005)
  - MCF: Browning (1976), Kleven-Kreiner (2006)
  - OT: Ramsey (1927), Diamond-Mirrlees (1971), Saez (2001)

Fraction of NBER Working Papers in Public Economics Using Sufficient Statistics Terminology

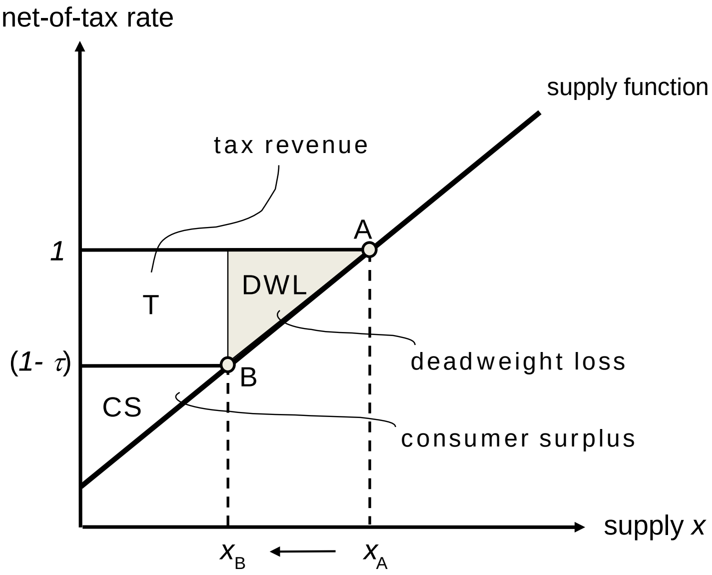


Source: Kleven (2018). “Language Trends in Public Economics”

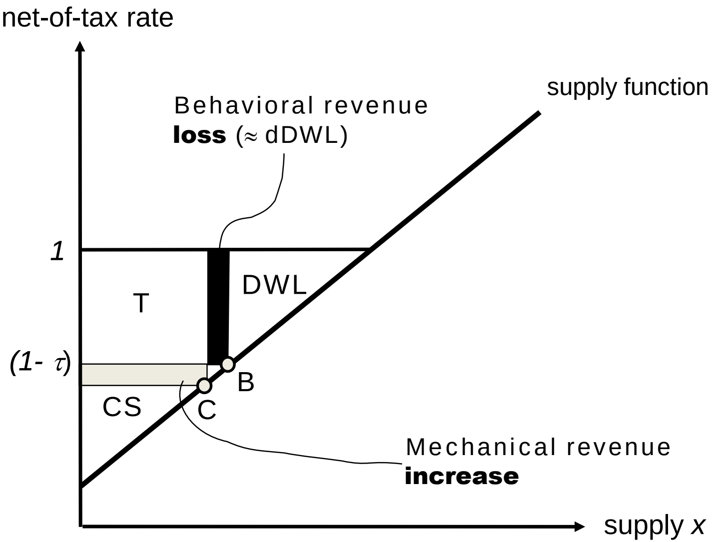
Outline

- Harberger Triangles
- A standard sufficient statistics approach:
  - Assume small reforms and no non-government distortions
  - Provide a general formula and Harberger-style formulas
  - We use the language of taxation, but the analysis is more general
- Link between sufficient statistics approach and optimal taxation
- Generalizations of the approach:
  - Large reforms
  - Non-government externalities or internalities

Harberger Triangle: Total DWL



Harberger Triangle: Marginal DWL



Marginal DWL Formula

- From the graphical analysis, we have

$$dDWL \approx -\tau \cdot dx = \frac{\tau}{1-\tau} \cdot x \cdot \varepsilon \cdot d\tau,$$

  where $\varepsilon \equiv \frac{dx/x}{d(1-\tau)/(1-\tau)}$ is the elasticity of supply x with respect to the net-of-tax rate $(1-\tau)$

- Conditional on the observables $(x, \tau, d\tau)$, the reduced-form elasticity ε is a "sufficient statistic" for welfare evaluation

- How general is this argument?

The Basic Principle

- Define $DWL = W - T$ where W is a money-metric utility loss from taxation and T is tax revenue
- The marginal DWL from tax reform is $dDWL = dW - dT$
  - We have $dT = dM + dB$, where dM is the **mechanical effect** and dB is the **behavioral effect**
  - Assuming small reform and no non-government externalities, we have $dW = dM$ (**envelope theorem**)
  - This implies $dDWL = dW - dT = dM - (dM + dB) = -dB$
- Hence, the marginal efficiency cost equals the behavioral revenue loss (**fiscal externality**)

A Formal Model

- Utility of individual i:

$$u^i\left(x_0^i, \ldots, x_J^i\right),$$

where $j = 0, ..., J$ index different goods (demands and supplies) as well as time

- Budget constraint

$$\sum_{j=0}^{J} x_j^i + T\left(x_0^i, \ldots, x_J^i\right) = y^i \quad \Leftrightarrow \quad \sum_{j=0}^{J}\left(1+\tau_j^i\right) x_j^i = Y^i,$$

where the piecewise linear function $T(\cdot)$ embodies all taxes and transfers, $\tau_j^i \equiv \partial T / \partial x_j^i$ are marginal tax rates, and $Y^i \equiv y^i + \sum_{j=0}^{J} \tau_j^i x_j^i - T\left(x_0^i, ..., x_J^i\right)$ is virtual income

Individual Optimization

- FOC for good j:

$$\frac{\partial u^i}{\partial x_j^i} - \lambda^i \left(1 + \tau_j^i\right) = 0$$

- Indirect utility: $v^i \left(1 + \tau_0^i, \ldots, 1 + \tau_J^i, Y^i\right)$

- Derivatives of indirect utility (**envelope conditions**):

$$\frac{\partial v^i}{\partial Y^i} = \lambda^i, \qquad \frac{\partial v^i}{\partial \left(1 + \tau_k^i\right)} = -\lambda^i x_k^i \tag{1}$$

Utility Effect of Small Reforms

- Specify tax policy in terms of a treatment parameter θ, i.e. $T\left(x_0^i, \ldots, x_J^i, \theta\right)$ and $\tau_j^i(\theta) \quad \forall j$

- Changes in θ may capture any changes in $\tau_0^i, \ldots, \tau_J^i$ and $T(\cdot)$. For now we focus on small reforms, $d\theta \approx 0$

- Using the envelope conditions (1) and the definition of Y^i:

$$\frac{dv^i/d\theta}{\lambda^i} = -\frac{\partial T^i}{\partial \theta}$$

  $\rightarrow$ The utility effect of any arbitrary, small reform equals the mechanical revenue effect

From Individual Welfare to Social Welfare

- Social welfare function:

$$W(\theta) = \int_i \omega^i v^i(\theta)\, di + \mu \int_i T^i(\theta)\, di$$

where ω^i is a Pareto weight on individual i and μ is the marginal value of government revenue

- Social welfare effect of reform:

$$\frac{dW/d\theta}{\mu} = \int_i \left[\underbrace{\left(\frac{dT^i}{d\theta} - \frac{\partial T^i}{\partial \theta} \right)}_{\text{efficiency}} + \underbrace{\left(1 - g^i\right) \frac{\partial T^i}{\partial \theta}}_{\text{equity}} \right] di$$

where $g^i \equiv \frac{\omega^i \lambda^i}{\mu}$ is the social welfare weight on individual i

Efficiency Effect = Fiscal Externality

The effect of any small tax reform on economic efficiency equals

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \int_i \left[\frac{dT^i}{d\theta} - \frac{\partial T^i}{\partial \theta}\right] di,$$

i.e., the difference between the total and mechanical revenue effects, corresponding to the behavioral revenue effect = fiscal externality

Sufficient Statistics Formula

The effect of any small reform on economic efficiency can be written in terms of "reduced-form" elasticities:

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \int_i \sum_{j=0}^{J} \left[\sum_{k=0}^{J} \tau_j^i x_j^i \varepsilon_{jk}^i \frac{d\tau_k^i/d\theta}{1+\tau_k^i} - \tau_j^i x_j^i \eta_j^i \frac{\partial T^i/\partial\theta}{Y^i} \right] di,$$

where ε_{jk}^i is the Hicksian price elasticity of good j wrt. the price on good k, and η_j^i is the income elasticity of good j

- The sufficient statistics for evaluating reform are $\left\{\varepsilon_{jk}^i, \eta_j^i\right\}_{\forall j,k,i}$

Comments on Sufficient Statistics Formula

- The formula accounts for **general equilibrium effects**
  - It accounts for distortions in all markets and allows for any possible cross-effects between markets
  - Explicitly accounting for pre-tax price changes (tax incidence) will not change the formula
- The formula accounts for **dynamics**
  - Indices j, k capture any time dimension
- But the parameter space is too large for empirical implementation $\rightarrow$ impose more structure by (i) restricting policy space and/or (ii) restricting behavioral responses

Harberger-Style Formula

- Assumptions:
  1. Utility is quasi-linear ($\eta_j^i = 0 \;\; \forall j, i$)
  2. Only one good is taxed (e.g. good 0)
  3. Linear tax (marginal rate τ_0)

- We then obtain:

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \bar{\varepsilon}_0 \cdot \frac{\tau_0}{1+\tau_0} \cdot \frac{d\tau_0}{d\theta},$$

where $\bar{\varepsilon}_0 \equiv \int_i \left[x_0^i \varepsilon_{00}^i\right] di$ is the demand-weighted average Hicksian elasticity in the population

Harberger-Style Formula Generalized

- Relaxed assumptions:
    1. Utility is quasi-linear $(\eta_j^i = 0 \;\; \forall j, i)$
    2. Goods $0, ..., J_0$ are taxed at rate τ_0 and goods $J_0 + 1, ..., J$ are taxed at rate τ_1 $\rightarrow$ normalize $\tau_1 = 0$

- We then obtain the same formula:

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \bar{\varepsilon}_0 \cdot \frac{\tau_0}{1+\tau_0} \cdot \frac{d\tau_0}{d\theta},$$

but where $\bar{\varepsilon}_0 \equiv \int_i \left[\sum_{j=0}^{J_0} \sum_{k=0}^{J_0} x_j^i \varepsilon_{jk}^i\right] di$ is a demand-weighted average Hicksian elasticity across goods $0, ..., J_0$ with respect to the tax price $1 + \tau_0$ on all those goods

Labor Supply Interpretation

- Goods $0, ..., J_0$ are labor supplies in different periods; goods $J_0 + 1, ..., J$ are consumption in different periods
- The sufficient statistic $\bar{\varepsilon}_0$ is a **lifetime earnings-weighted elasticity of labor supply wrt a permanent labor tax**
- If $0, ..., J_0$ also include multi-dimensional earnings components, $\bar{\varepsilon}_0$ is a **lifetime earnings-weighted elasticity of taxable income wrt permanent labor tax** (generalizes Feldstein 1999)
- Uniform tax on consumption in different periods rules out capital taxes. To allow for capital taxes and still have just one sufficient statistic $\bar{\varepsilon}_0$, we have to add separability assumptions between labor and consumption

Link to Optimal Taxation

- For optimal taxation, we must have $dW/d\theta = 0$ for any θ, i.e.

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \int_i \left(g^i - 1\right) \frac{\partial T^i}{\partial \theta} di$$

  where LHS is the efficiency effect and RHS is the equity effect

- In the Harberger-style special case, this implies

$$\frac{\tau_0}{1+\tau_0} = \frac{\int_i \left(g^i - 1\right) x_0^i di}{\bar{\varepsilon}_0}$$

- It is tempting to call $\bar{\varepsilon}_0$ a sufficient statistic for optimal taxation, but this is now the elasticity *at the optimal point* $\rightarrow$ global knowledge of demand functions $\rightarrow$ structural approach

Generalizations

We will generalize the approach by relaxing its two key assumptions:

1. Allow for large reforms
2. Allow for non-government externalities and internalities
    - Some sufficient statistics papers (e.g. Piketty et al. 2014) model specific market imperfections, but we develop a general approach

Welfare Effect of Large Reforms

- Specify marginal tax rates as $\tau_j^i + \theta \Delta \tau_j^i$ where $\Delta \tau_j^i$ is the reform-induced tax rate change $\rightarrow$ pre-reform policy corresponds to $\theta_0 = 0$ and the post-reform policy corresponds to $\theta_1 = 1$

- An exact welfare formula:

$$\Delta W = W(1) - W(0) = \int_0^1 \frac{dW}{d\theta} d\theta$$

- Trapezoid approximation of welfare formula:

$$\Delta W \approx \frac{1}{2} \left\{ \frac{dW(0)}{d\theta} + \frac{dW(1)}{d\theta} \right\}$$

Sufficient Statistics for Large Reforms: Trapezoid

Assuming quasi-linear utility and a single tax rate τ_0 on taxed goods, the efficiency effect of large reforms can be approximated as

$$\left.\frac{\Delta W}{\mu_0}\right|_{g^i=1} \approx \bar{\varepsilon}_0 \cdot \frac{\tau_0}{1+\tau_0} \cdot \Delta\tau_0 + \frac{1}{2}\left\{\bar{\varepsilon}_0 \cdot \Delta\left[\frac{\tau_0}{1+\tau_0}\right] + \Delta\bar{\varepsilon}_0 \cdot \frac{\tau_0}{1+\tau_0} + \Delta\bar{\varepsilon}_0 \cdot \Delta\left[\frac{\tau_0}{1+\tau_0}\right]\right\} \cdot \Delta\tau_0,$$

where $\bar{\varepsilon}_0$ is the elasticity at the initial policy and $\Delta\bar{\varepsilon}_0$ is the elasticity change due to the policy

- The sufficient statistics are the elasticity level $\bar{\varepsilon}_0$ and the elasticity change $\Delta\bar{\varepsilon}_0$

Sufficient Statistics for Large Reforms: Iso-Elastic

Assuming quasi-linear, iso-elastic utility and a single tax rate τ_0 on taxed goods, the efficiency effect of large reforms can be approximated as

$$\left.\frac{\Delta W}{\mu}\right|_{g^i=1} \approx \bar{\varepsilon}_0 \cdot \left\{ \frac{\tau_0}{1+\tau_0} + \frac{1}{2}\Delta\left[\frac{\tau_0}{1+\tau_0}\right] \right\} \cdot \Delta\tau_0,$$

a standard Harberger-style formula using a modified tax wedge, $\frac{\tau_0}{1+\tau_0} + \frac{1}{2}\Delta\left[\frac{\tau_0}{1+\tau_0}\right]$

Sufficient Statistics for Large Reforms: Conclusions

- In general, we have to account for the entire path of wedges and elasticities between the pre-reform and post-reform equilibria
- We can provide approximations that depend on the changes in wedges and elasticities from before to after the reform
  - Sufficient statistics include elasticity levels and elasticity changes
  - Trapezoid and 2nd-order Taylor appr. give similar results
- If elasticity changes cannot be estimated, we have to impose more structure
  - The simplest solution is to assume iso-elastic utility $\rightarrow$ a fully structural approach

Welfare Effect with Non-Government Distortions

- Utility of individual i:

$$u^i\left(x_0^i, ..., x_J^i; E_0^i, ..., E_J^i\right),$$

where E_j^i is the externality on individual i due to the consumption of good j:

$$E_j^i = \int_{\hat{\imath}} \phi_j^{i\hat{\imath}} x_j^{\hat{\imath}} d\hat{\imath},$$

where $\phi_j^{i\hat{\imath}}$ captures the externality that individual $\hat{\imath}$ imposes on individual i when consuming good j

- Different assumptions on $\phi_j^{i\hat{\imath}}$ capture a wide range of cases, e.g. atmospheric externalities, relative consumption or relative earnings externalities, and internalities

Fiscal Externality and Non-Government Externalities

In the presence of non-government externalities, the effect of any small tax reform on economic efficiency is given by

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \int_i \left[\frac{dT^i}{d\theta} - \frac{\partial T^i}{\partial \theta} + \sum_{j=0}^{J} \frac{\partial v^i/\partial E_j^i}{\lambda^i}\frac{dE_j^i}{d\theta}\right] di,$$

where the first term is the fiscal externality and the second term includes any non-government externalities

- The additivity between tax distortions and non-tax externalities echoes Sandmo (1975) for the case of atmospheric externalities, but here we have taken a more general approach

Simplifying Assumptions

To obtain simple sufficient statistics results, we make two assumptions:

1. Demands and supplies are independent of the external effects, i.e. $x_j^i = x_j^i \left(1 + \tau_0^i, ..., 1 + \tau_J^i, Y^i\right)$
2. The externality parameter equals $\phi_j^{i\hat{\imath}} = \phi_{I_j}^i \cdot \mathbf{1}\left(\hat{\imath} = i\right) + \phi_{E_j}^i$, where $\phi_{I_j}^i$ is an internality and $\phi_{E_j}^i$ is an externality
   - Different combinations of $\phi_{I_j}^i$ and $\phi_{E_j}^i$ encompass all of the examples provided earlier

Sufficient Statistics with Non-Government Externalities

In the presence of non-government externalities, the effect of any small tax reform on economic efficiency can be written as

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \int_i \left[\sum_{j=0}^{J} \sum_{k=0}^{J} \hat{\tau}_j^i x_j^i \varepsilon_{jk}^i \frac{d\tau_k^i/d\theta}{1+\tau_k^i} - \sum_{j=0}^{J} \hat{\tau}_j^i x_j^i \eta_j^i \frac{\partial T^i/\partial\theta}{Y^i} \right] di,$$

where $\hat{\tau}_j^i \equiv \tau_j^i + \tau_{I_j}^i + \tau_{E_j}$ is an externality-adjusted tax wedge on good j for individual i

- The sufficient statistics are $\left\{\varepsilon_{jk}^i, \eta_j^i, \hat{\tau}_j^i\right\}_{\forall j,k,i}$

Harberger-style special case

- Assumptions:
  1. Utility is quasi-linear ($\eta_j^i = 0 \; \forall j, i$)
  2. Only good 0 is distorted ($\hat{\tau}_j^i = 0$ for $j \geq 1$)
  3. Both the tax and non-tax distortions are homogeneous across individuals ($\tau_0^i = \tau_0$ and $\hat{\tau}_0^i = \hat{\tau}_0$)

- We then obtain:

$$\left.\frac{dW/d\theta}{\mu}\right|_{g^i=1} = \bar{\varepsilon}_0 \cdot \frac{\hat{\tau}_0}{1+\tau_0} \cdot \frac{d\tau_0}{d\theta},$$

where $\bar{\varepsilon}_0 \equiv \int_i \left[x_0^i \varepsilon_{00}^i\right] di$

- The sufficient statistics are $\bar{\varepsilon}_0$ and $\hat{\tau}_0$

Sufficient Statistics with Non-Government Externalities

- Under a general formulation, we have developed sufficient statistics formulas that retain the standard form:
  - Interactions between elasticities and modified "tax wedges"
  - The modified wedges include any externality/internality effects
  - Welfare evaluation does not require a fully specified model of each market imperfection, but can be written in terms of reduced-form gaps between private and social prices
- The empirical challenge is that we have to estimate both the elasticities and the externality-adjusted wedges