Calculating Reduced-Form Elasticities Using Notches

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Abstract

This technical note clarifies how to calculate reduced-form elasticities based on estimates of behavioral responses obtained from bunching at notches. It derives two trapezoid approximations that complement the simple upper-bound elasticity formula provided in Kleven & Waseem (2013), and it clarifies how to calculate the reduced-form elasticities reported in that paper.

Kleven & Waseem (2013) develop two approaches for calculating elasticities using bunching responses to notches: an exact structural approach (equation 5) and a reduced-form approximation (equation 12). As described in Kleven & Waseem (2013), the simple reduced-form formula is not an approximation of the true elasticity, but rather an *upper bound* on the true elasticity. In this note, I provide alternative reduced-form formulas that *are* approximations of the true elasticity.

While the numerator of the reduced-form elasticity is conceptually straightforward (the behavioral response $\Delta z^*/z^*$ estimated from bunching), it is less obvious what is the appropriate denominator given the incentive that drives the response is a jump in the *average* tax rate. The idea of the reduced-form approximation is to translate the notch into a hypothetical kink — a discrete jump in the *marginal* tax rate — that would have created the same response as the one observed at the notch. The Kleven-Waseem reduced-form formula (illustrated in their Figure III) relates the behavioral response $\Delta z^*/z^*$ to the change in the implicit marginal tax rate between z^* and $z^* + \Delta z^*$ created by the notch. As illustrated in Figure III, a kink of that size would not be sufficient to generate the response $\Delta z^*/z^*$ (the denominator is "too small"), which is why the formula is an upper bound.

To obtain a more precise approximation, note that the kink needed to generate the observed response is such that the upper bracket is tangent to the indifference curve of the marginal buncher at the threshold z^*. Let us denote this (unknown) slope by $1 - \tau^*$. The exercise is to approximate $1 - \tau^*$ using the trapezoid rule. For the purpose of this approximation, we will assume that the notch is small in terms of its interior incentives, i.e. $z^I \approx z^* + \Delta z^*$. That is, if the marginal buncher were to stay in the interior, this individual would not move very far away from the original location. With this assumption and denoting the indifference curve in (z, c) space by $I(z)$, we have

$$\frac{\int_{z^*}^{z^*+\Delta z^*} I'(z)\, dz}{\Delta z^*} \approx \frac{I'(z^*) + I'(z^* + \Delta z^*)}{2} = \frac{(1-\tau^*) + (1-t-\Delta t)}{2}, \qquad (1)$$

where we use the standard trapezoid rule. The last equality uses that the slope at z^* has been defined as $1 - \tau^*$, and that the slope at $z^* + \Delta z^*$ (given this is the best interior point) equals $1 - t - \Delta t$.

We have expressed the average slope of the indifference curve on $(z^*, z^* + \Delta z^*)$ as a function of the unknown entity, $1-\tau^*$. We can now use that we know the average slope of the indifference curve on this interval, because this is simply the implicit tax rate $1 - t^*$ on $(z^*, z^* + \Delta z^*)$ as defined in Kleven & Waseem (2013, eq. 11). There we have

$$t^* \equiv \frac{T(z^* + \Delta z^*) - T(z^*)}{\Delta z^*} = t + \frac{\Delta t \cdot (z^* + \Delta z^*)}{\Delta z^*} \approx t + \frac{\Delta t \cdot z^*}{\Delta z^*}, \tag{2}$$

where the last approximation requires that the notch is small ($\Delta t \approx 0$). Combining equations (1)-(2), we obtain

$$1 - t^* \approx \frac{(1 - \tau^*) + (1 - t - \Delta t)}{2} \quad \Leftrightarrow \quad \tau^* \approx t + 2 \cdot \frac{\Delta t \cdot z^*}{\Delta z^*}, \tag{3}$$

where again we assume that the notch is small ($\Delta t \approx 0$).

The reduced-form elasticity with respect to the marginal net-of-tax rate $1 - \tau^*$ can now be approximated as

$$e_R \equiv \frac{\Delta z^* / z^*}{\Delta \tau^* / (1 - \tau^*)} \approx \frac{1}{2} \cdot \frac{(\Delta z^* / z^*)^2}{\Delta t / (1 - t)}, \tag{4}$$

which is one-half of the upper bound on e_R provided in Kleven & Waseem (2013).

Alternatively, we can derive a slightly more accurate (but clunkier) elasticity formula by using the exact expression for t^* — i.e., $t^* = t + \Delta t (z^* + \Delta z^*) / \Delta z^*$ — rather than the approximated expression in the last step of equation (2). In this case, we have

$$\tau^* \approx t + (2 + \Delta z^* / z^*) \cdot \frac{\Delta t \cdot z^*}{\Delta z^*},$$

in which case the reduced-form elasticity with respect to the marginal net-of-tax rate $1 - \tau^*$ is equal to

$$e_R \equiv \frac{\Delta z^* / z^*}{\Delta \tau^* / (1 - \tau^*)} \approx \frac{1}{2 + \Delta z^* / z^*} \cdot \frac{(\Delta z^* / z^*)^2}{\Delta t / (1 - t)}. \tag{5}$$

This is the formula that underlies the reduced-form elasticity estimates provided in Kleven & Waseem (2013), specifically in Tables II-III, columns (10)-(11). The reduced-form elasticity estimates provided in these two tables can be reconstructed based on the statistics in the tables and the expression for e_R given in equation (5) above.

In most practical applications, the difference in estimates between equation (5) and the simpler equation (4) will be tiny. Of course, the difference between both of these refinements and the upper-bound formula described in Kleven & Waseem (2013, eq. 12) equals a factor of about two.